Date ___/___/_____

Eg → FV → $1000, CR → 10% payable annually, n = 5 years, Price → $957.

Let's calculate Ytm → 11.17%.

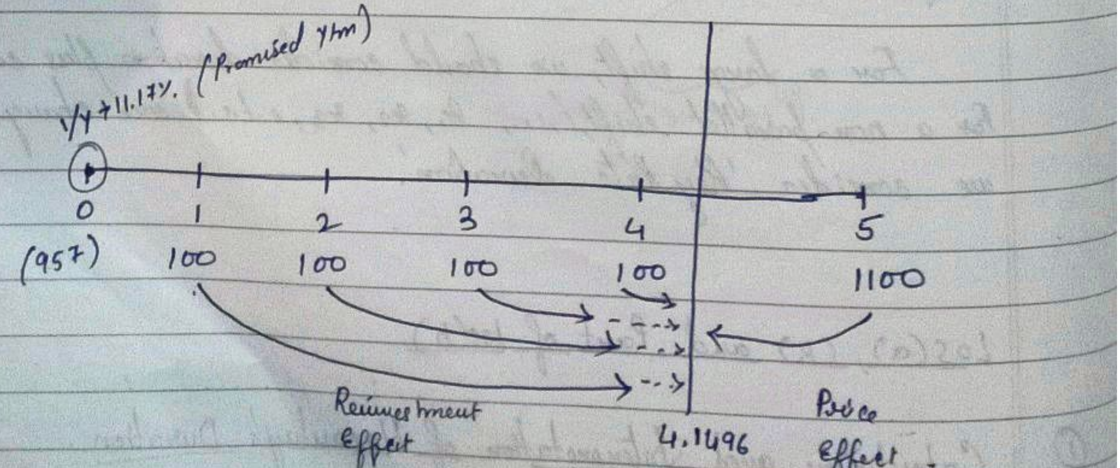
Years (x)	Cashflow	PV of CF @ 11.17	w	wx
1	100	89.95	0.094	0.094 → KRD_1
2	100	80.91	0.0846	0.1692 → KRD_2
3	100	72.78	0.0761	0.2283 → KRD_3
4	100	65.47	0.0684	0.2736 → KRD_4
5	1,100	644.82	0.6769	3.3845 → KRD_5
		957.00	1	4.1496

Macaulay's Duration (D) = 4.1496.

Interpretation → ① It is the sensitivity of a bond with respect to a change in 'continuous yield' (not testable here on the term continuous)

② It is the average Waiting time. (Not Imp).

③ It is the immunising period, i.e., the Holding Period for which, 'Reinvestment Effect' and 'Price Effect', exactly cancel out each other, such that, investor's realised yield equals promised 'ytm'.



Page No.

Date ___/___/_____

$\Rightarrow \quad 957 = \dfrac{1485.04}{(1+r)^{4.1496}}.$

OR

$\rightarrow$ 2nd clr TVM.

957 ± PV

1485.04 - FV

4.1496 - n

CPT - I/Y $\Rightarrow$ 11.17%. (Proved)

Case ② → Interest Rates in the Market falls to 10%, just before the 1st Coupon.

Step 1 → Future Value of Reinvested Coupon:

100 - PMT.

4 - n

I/Y - 10

CPT - FV $\Rightarrow$ \$464.10.

464.10 - PV

0.1496 - n

10 - I/Y

CPT - FV $\Rightarrow$ ~~\$466.94~~ 470.76

400

Coupon

= 100 × 4

~~70.76~~

Interest on Interest.

This is lower than what would have been achieved if reinvestment rate remained @ 11.17%.

Step 2 → Selling Price at t = 4.1496.

$\Rightarrow \dfrac{1100}{(1.10)^{0.8504}}$ $\Rightarrow$ 1014.36 $\Rightarrow$ This is more than the 'Cons

Page No.

Date ___/___/___

Proof: Case ① Interest Rate in the Market rises to 12% just before the 1st Coupon.

- Future Value of Reinvestment Coupon

$$\Rightarrow 100(1.12)^{3.1496} + 100(1.12)^{2.1496} + 100(1.12)^{1.1496} + 100(1.12)^{0.1496}$$

$$= \$486.10.$$

OR

100 - PMT

4 - n

12 - I/Y

CPT - FV $\Rightarrow$ 477.93.

2nd Clr TVM.

477.93 - PV

0.1496 - n

12 - I/Y

CPT $\Rightarrow$ FV $\Rightarrow$ \$486.10.

Coupon $= 100 \times 4 = 400$

Interest on Interest $= 86.10$

$\Downarrow$ This is higher than what would have been achieved at a reinvestment rate of 11.17%.

Selling Price at $t = 4.1496$.

$$\Rightarrow \frac{1100}{(1.12)^{0.8504}}$$

$\Rightarrow$ \$998.94. $\Rightarrow$ This is less than the price as per 'constant yield price trajectory'.

Total Cashflow $\Rightarrow$ (486.10 + 998.94)

= \$1485.04.

Hence, Annualised Holding Return, i.e., realised yield is given by Outflow = Inflow.

Page No.

Date ___/___/_____

Sol→ C, B, A, D. Note→ Bonds selling at premium has the highest coupon rate & hence, highest re-investment rate risk.

Capital Gain/loss

First of all, let us understand that it is arising out of Market Price risk, i.e., the risk of interest rate rising & bond ~~prices~~ price falling.

Thus, re-investment risk is a compounding effect (kaash int. rate barhe!), while Price risk is a discounting effect (kaash int. rate gire!). A category of investors, namely passive believe that, interest rate forecasting is impossible (God won't listen to our 'kaash'!). So, they want to earn the promised ytm. - they are advised to hold a bond for it's duration (not maturity)

Ofcourse, to calculate Capital Gain/loss, compare Selling Price, say $998.94 in case ① with the pulled to Par price, i.e., $\frac{1100}{(1.1177)^{0.8504}}$

$= \$1005.27$. - a capital loss of $(1005.27 - 998.94)$
$= \$6.33$.

Eg Consider the following bond:

FV → $1000
CR → 8%.
N → 3 years
Price → $960.

Calculate its Macaulay's duration

Sol First of all, calculate 'ytm'.
→ 9.6%.

Page No.

Date ___/___/___

Yield Price Trajectory'.

Total Cashflow = \$ (470.76 + 1014.36)
= \$ ~~1480.3~~ . 1485.12

Hence Annualised Holding Return, i.e., realised yield is given by

Outflow = Inflow.

$$957 = \cancel{1480}\ \frac{1485}{(1+r)^{4.1446}}.$$

Limitation → This immunisation only works for a one time parallel shift in the yield curve just prior to the first coupon.

Three sources of 'bond return' –
- Coupon
- Re-investment Income, i.e., Interest on Interest.
- Capital Gain/loss.

Let's elaborate into the 2nd & 3rd Source.

Reinvestment Income

'Reinvestment Risk', is the risk of interest rate falling & the investor forced to ~~re~~ re-invest at a lower rate.

Eg → Rank the following bonds in the Descending order of Reinvestment Risk.

A	B	C	D
5 yr 10% Coupon Straight Bond.	5 yr 14% Coupon Straight Bond	5 yr 10% Coupon Callable Bond	5 yr ZCB Straight Bond.

Date __/__/__

(b) Longer Holding Period (HP > D)

⇓

Reinvestment Effect > Price Effect

⇓

Reach interest rate rise [RY > Ytm].

vice versa.

(c) Holding Period = Duration

⇓

Reinvestment Effect = Price Effect.

(HPR) ⇓

∴ RY = Ytm.

Note → The text defines 'duration gap' = D − H. So, a positive gap means 'shorter holding period' & negative gap means longer holding period.

524 - Q - 1-6, 8, 31, 41.

pg - 552

LOS(b) → Types of Duration.

Q1 What is the equation for 'Bond Price'?

$$P_0 = \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \ldots \frac{C_n}{(1+r)^n}.$$

Q2 What is our objective in this LOS?

Sol → Assuming ~~linear~~ linear relation between P & R r, (in other words ignoring convexity), we want to find out the rate of change of P with respect to 'r'. This answer is called duration. However, there are 3 types of duration -

- Macaulay (D)
- Modified (MD)

Date ___/___/___

Step 2

Yrs (x)	CF	PV	w	wx	
1	80	72.99	0.076	0.076	→ KRD1
2	80	66.59	0.0694	0.1388	→ KRD2
3	1080	820.34	0.8546	2.5638	→ KRD3.
		$960	1	2.7786	

↓ Average Waiting Time

↓ Rate of Change of Price w.r.t. continuous yield

↓ Immunising Period.

⇓

Price Effect = Reinvestment Effect

⇓

Realised Yield = Promised Ytm.

Certain Properties

The bond is held till maturity and all cashflows received on time:-

(a) Interest rate doesnot change - Realised yield = Promised Ytm.

(b) Interest rate goes up - Realised Yield > Ytm.

(c) Interest rate falls - Realised Yield < Ytm.

If investor sells bond prior to maturity and there is one time parallel shift in yield, just prior to the first coupon -

(a) Shorter Holding Period (Holding Period < Duration)

⇓

Price Effect > Re-investment Effect.

⇓

Kaash interest rate falls [RY > Ytm]

vice versa.

Page No.