CA FINAL

# STRATEGIC FINANCIAL MANAGEMENT

## CHALLENGER SERIES

### Portfolio Management

# PROBLEM – 1

Mr. Dinesh Gupta has gathered the following information relating to six stocks. The risk free rate is 7%, return on the market index is 12% and variance of the return on the index is $25\%^2$.

| Stock | Alpha (%) | Unsystematic risk $(\%)^2$ | Total Risk $(\%)^2$ |
|---|---|---|---|
| BHEL | 2.10 | 45 | 75 |
| HLL | 1.25 | 15 | 40 |
| Cipla | 1.30 | 14 | 42 |
| HDFC | 1.45 | 20 | 55 |
| ACC | 0.95 | 14 | 32 |
| L&T | 0.82 | 16 | 35 |

You are **required** to

Construct a portfolio using Sharpe’s portfolio optimization model.

# Solution :

To calculate the expected return, beta of each stock should be estimated.

This can be calculated using the following relation:

Systematic risk = Total risk – Unsystematic risk = $\beta^2\sigma_m^2$

$$\beta_i = \left[\frac{\text{Systematic Risk}}{\sigma_m^2}\right]^{1/2}$$

| Stock | Systematic risk (%) $^2$ | $\beta_i = \left(\frac{\text{Systematic risk}}{\sigma_m^2}\right)^{1/2}$ |
|---|---|---|
| BHEL | 30 | $(30/25)^{1/2}$ = 1.095 |
| HLL | 25 | $(25/25)^{1/2}$ = 1.000 |
| Cipla | 28 | $(28/25)^{1/2}$ = 1.058 |
| HDFC | 35 | $(35/25)^{1/2}$ = 1.183 |
| ACC | 18 | $(18/25)^{1/2}$ = 0.849 |
| L&T | 19 | $(19/25)^{1/2}$ = 0.872 |

| Stock | Expected return $R_i = (\alpha+\beta R_m)$ | $\beta_i$ | $(R_i - R_f)$ | $\frac{(R_i - R_f)}{\beta_i}$ | Rank |
|---|---|---|---|---|---|
| BHEL | 15.240 | 1.095 | 8.240 | 7.525 | 1 |
| HLL | 13.250 | 1 | 6.250 | 6.250 | 4 |
| Cipla | 14.000 | 1.058 | 7.000 | 6.616 | 3 |
| HDFC | 15.646 | 1.183 | 8.646 | 7.309 | 2 |
| ACC | 11.138 | 0.849 | 4.138 | 4.874 | 5 |
| L&T | 11.284 | 0.872 | 4.284 | 4.913 | 6 |

| Rank | Security | $\beta_i$ | $\sigma_{ei}^2$ | $(R_i - R_f)$ | $\frac{(R_i - R_f)\beta_i}{\sigma_{ei}^2}$ | $\frac{\beta_i^2}{\sigma_{ei}^2}$ |
|---|---|---|---|---|---|---|
| 1 | BHEL | 1.095 | 45 | 8.240 | 0.200 | 0.0270 |
| 2 | HDFC | 1.183 | 20 | 8.646 | 0.511 | 0.0700 |
| 3 | Cipla | 1.058 | 14 | 7.000 | 0.529 | 0.0800 |
| 4 | HLL | 1 | 15 | 6.250 | 0.417 | 0.0670 |
| 5 | L&T | 0.872 | 16 | 4.284 | 0.233 | 0.0475 |
| 6 | ACC | 0.849 | 14 | 4.138 | 0.251 | 0.0515 |

| $\sum_{i=1}^{i}\frac{(R_i-R_f)\beta_i}{\sigma_{ei}^2}$ | $\sum_{i=1}^{i}\frac{\beta_i^2}{\sigma_{ei}^2}$ | $1+\sigma_m^2\sum_{i=1}^{i}\frac{\beta_i^2}{\sigma_{ei}^2}$ | $\sigma_m^2\sum_{i=1}^{i}\left(\frac{R_i-R_f}{\sigma_{ei}^2}\right)\beta_i$ | C |
|---|---|---|---|---|
| 0.200 | 0.027 | 1.675 | 5.000 | 2.985 |
| 0.711 | 0..097 | 3.425 | 17.775 | 5.189 |
| 1.240 | 0.177 | 5.425 | 31.000 | 5.714 |
| 1.657 | 0.244 | 7.100 | 41.425 | 5.834* |
| 1.890 | 0.291 | 8.280 | 47.250 | 5.706 |
| 2.141 | 0.343 | 9.575 | 53.525 | 5.590 |

Where

$$C = \frac{\sigma_m^2\sum_{i=1}^{i}\frac{(R_i-R_f)}{\sigma_{ei}^2}}{1+\sigma_m^2\sum_{i=1}^{i}\frac{\beta_i^2}{\sigma_{ei}^2}}$$

$$Z_i = \frac{\beta_i}{\sigma_{ei}^2}\left[\frac{R_i-R_f}{\beta_i}-c^*\right]$$

$$Z_1 = \frac{1.095}{45}[7.525-5.834] = 0.0411$$

$$Z_2 = \frac{1.095}{45}[7.309-5.834] = 0.0872$$

$$Z_3 = \frac{1.058}{14}[6.616-5.834] = 0.0591$$

$$Z_4 = \frac{1}{15}[6.25-5.834] = 0.0277$$

$$\sum_{i=1}^{i} Z_i = 0.2151$$

| Security | Proportions | | |
|---|---|---|---|
| BHEL | $\frac{0.0411}{0.2151}$ = | 0.1910 | 19.10% |
| HDFC | $\frac{0.0872}{0.2151}$ = | 0.4054 | 40.54% |
| Cipla | $\frac{0.0591}{0.2151}$ = | 0.2748 | 27.48% |
| HLL | $\frac{0.0277}{0.2151}$ = | 0.1288 | 12.88% |

# PROBLEM – 2

A market analyst has estimated probable returns under different macroeconomic conditions of the following three stocks:

| Stock | Current price (Rs.) | Rate of return (%) | | |
|---|---|---|---|---|
| | | Recession | Moderate Growth | Boom |
| Polar Informatics | 18 | -15 | 10 | 25 |
| Season Biotech | 20 | 15 | 8 | -4 |
| Good Value Ltd. | 76 | 17 | 20 | 12 |

He is exploring if it is possible to make any arbitrage profits from the above information.

**Required:**

Using the above information construct an arbitrage portfolio and show the payoffs under different economic scenarios.

# Solution :

Arbitrage portfolio can be constructed by selling 2 stock of Polar Industries and Shania biotech and buying one stock of Good value Ltd.

Market price of the stocks under different scenarios.

| | Recession | Moderate Growth | Boom |
|---|---|---|---|
| Polar Informatics | 15.30 | 19.80 | 22.50 |
| Season Biotech | 23.00 | 21.60 | 19.20 |
| Good Value Ltd. | 88.92 | 91.20 | 85.12 |

Return

| | Recession | Moderate Growth | Boom |
|---|---|---|---|
| Sell 2 stock of Polar | $-15.30 \times 2$ | $-19.80 \times 2$ | $-22.50 \times 2$ |
| Sell 2 stock of season Biotech | $-23.00 \times 2$ | $-21.60 \times 2$ | $-19.20 \times 2$ |
| Buy one stock of Good value | 88.92 | 91.20 | 85.12 |
| | 12.32 | 8.40 | 1.72 |

# PROBLEM – 3

An investor holds the following stocks in his portfolio. All these stocks were purchased on March 1, 2002 at the following prices:

| Name of the company | No. of shares | Price per share |
|---|---|---|
| Sonata Cements | 1000 | Rs.28.00 |
| J. K. Cosmetics | 4000 | Rs. 7.00 |
| Prism Tyres | 2000 | Rs.14.00 |
| Sonal Software | 500 | Rs.56.00 |

The correlation coefficients of these stocks' return with the market and their standard deviation of returns were as follows:

| | Standard Deviation | Correlation Coefficient with the market index |
|---|---|---|
| Sonata Cements | 20% | 0.95 |
| J. K. Cosmetics | 18% | 0.85 |
| Prism Tyres | 14% | 0.72 |
| Sonal Software | 11% | 0.45 |

On February 28, 2003, the market prices and standard deviation of the returns of these four stocks were as follows:

| Name of the company | Price per share | Standard Deviation (%) |
|---|---|---|
| Sonata Cements | Rs.34.00 | 22.00% |
| J. K. Cosmetics | Rs. 5.50 | 17.00% |
| Prism Tyres | Rs.10.00 | 15.50% |
| Sonal Software | Rs.70.00 | 12.50% |

If the standard deviations of market's return is constant at 15% and assuming that correlation coefficients of stock's returns with the market index remains unchanged during last year, estimate the changes in the proportions of systematic and unsystematic risk of the portfolio.

(The covariance of returns between two stocks is product of their betas and market variance.)

# Solution :

**Total investment**

| | | |
|---|---|---|
| Sonata Cements | $1000 \times 28$ | 28000 |
| J. K. Cosmetics | $4000 \times 7.00$ | 28000 |
| Prism Tyres | $2000 \times 14.00$ | 28000 |
| Sonal Software | $500 \times 56.00$ | 28000 |
| | | 112000 |

Equal amount of investment is made in each of the stocks.

Beta of stocks = $\rho \frac{\sigma_i}{\sigma_m}$

20 Sonata Cements, $\beta_{SC} = 0.95 \times \frac{20}{15} = 1.267$

J. K. Cosmetics , $\beta_{JK} = 0.85 \times \frac{18}{15} = 1.02$

Prism Tyre, $\beta_{PT} = 0.72 \times \frac{14}{15} = 0.672$

Sonal Software, $\beta_{SS} = 0.45 \times \frac{11}{15} = 0.33$

Portfolio Beta, $\beta_P = \frac{1}{4}(1.267 + 1.02 + 0.672 + 0.33)$

$= 0.82225.$

**Systematic risk of the portfolio**

$= \beta_P^2 \sigma_m^2$

$= (0.8225)^2 \times (15)^2$

$= 152.03\ (\%)^2.$

To calculate two stocks we have to use the following formula

$Cov_{JK} = \beta_J \beta_K \sigma_m^2$

$Cov_{SJ} = 1.267 \times 1.02 \times 225 = 290.78(\%)^2$

$Cov_{BP} = 1.267 \times 0.672 \times 225 = 191.57\ (\%)^2$

$Cov_{SS} = 1.267 \times 0.33 \times 225 = 94.07(\%)^2$

$Cov_{JP} = 1.02 \times 0.672 \times 225 = 154.224\ (\%)^2$

$Cov_{JS} = 1.02 \times 0.33 \times 225 = 75.735\ (\%)^2$

$Cov_{PS} = 0.672 \times 0.33 \times 225 = 49.90(\%)^2$

**Total risk of the portfolio**

$$= (0.25)^2 \times (20)^2 + (0.25)^2 \times (18)^2 + (0.25)^2 \times (14)^2 + (0.25)^2 \times (11)^2 + \frac{1}{16}(2 \times 290.78 + 2 \times 191.57 + 2 \times 94.07 + 2 \times 154.224 + 2 \times 75.725 + 2 \times 49.90)$$

$$= 25 + 20.25 + 12.25 + 7.5625 + \frac{1}{16}(581.56 + 383.14 + 188.14 + 308.448 + 151.47 + 99.8)$$

$$= 65.0625 + 107.034 = 172.097\ (\%)^2$$

Unsystematic risk $= 172.097 - 152.03 = 20.067(\%)^2$

Proportion of Systematic risk $= \dfrac{152.03}{172.097} = 88.34\%$

Unsystematic risk $= \dfrac{20.067}{172.097} = 11.66\%.$

**After one year**

When the price of stock changes, the weightage of the stocks value also changes and as the S.D. of the stock changes their beta also changes.

**Investment value**

| | | |
|---|---|---|
| Sonata Cements | $34 \times 1000$ | 34000 |
| J.K. Cosmetics | $5.50 \times 4000$ | 22000 |
| Prism Tyres | $10 \times 2000$ | 20000 |
| Sonal Software | $70 \times 500$ | 35000 |
| | | 111000 |

New Weights of the stocks in the portfolio

| | | |
|---|---|---|
| Sonata Cements | 34000/111000 | 0.306 |
| J.K. Cosmetics | 22000/111000 | 0.198 |
| Prism Tyres | 20000/111000 | 0.18 |
| Sonal Software | 35000/111000 | 0.316 |

Beta of the stock after one year

| | | |
|---|---|---|
| Sonata Cements | $0.95 \times \frac{22}{15}$ | 1.393 |
| J.K. Cosmetics | $0.85 \times \frac{17}{15}$ | 0.9633 |
| Prism Tyres | $0.72 \times \frac{15.5}{15}$ | 0.794 |
| Sonal Software | $0.45 \times \frac{12.5}{15}$ | 0.375 |

New Covariances between Stock's return

$$\begin{aligned}
Cov_{SJ} &= 1.393 \times 0.9633 \times 225 &&= 301.922 \\
Cov_{SP} &= 1.393 \times 0.744 \times 225 &&= 227.137 \\
Cov_{SS} &= 1.393 \times 0.375 \times 225 &&= 397.8 \\
Cov_{JP} &= 0.9633 \times 0.744 \times 225 &&= 161.256 \\
Cov_{JS} &= 0.9633 \times 0.375 \times 225 &&= 81.278 \\
Cov_{PS} &= 0.744 \times 0.375 \times 225 &&= 62.775
\end{aligned}$$

New total risk of the portfolio

$$(0.306)^2 \times (22)^2 + (0.198)^2 \times (17)^2 + (0.18)^2 \times (15.5)^2 + (0.316)^2 \times (12.5)^2 + 2 \times 0.306 \times 0.198 \times 301.922 + 2 \times 0.306 \times 0.18 \times 227.137 + 2 \times 0.306 \times 0.316 \times 397.8 + 2 \times 0.198 \times 0.18 \times 116.256 + 2 \times 0.198 \times 0.316 \times 81.278 + 2 \times 0.186 \times 0.316 \times 62.775 = 244.41(\%)^2$$

Portfolio beta

= 0.306 × 1.393 × 0.9633 × 0.198 + 0.18 × 0.744 + 0.316 × 0.375 = 0.8694.

Systematic risk of the portfolio

$$= \beta_P^2 \sigma_m^2$$

= $(0.8694)^2 \times (15)^2$ = 170.067 $(\%)^2$.

Unsystematic risk = 170.067 – 244.41 = 74.34$(\%)^2$.

$$\text{Proportion of systematic risk} = \frac{(170.067)}{244.41} = 0.696 = 69.6\%.$$

$$\text{Unsystematic risk} = \frac{74.34}{244.41} = 0.304 = 30.4\%.$$

Clearly proportion of systematic has decreased from 88.34% to 69.6% and unsystematic risk has increased from 11.68% to 30.4%.

# PROBLEM – 4

The following data is related to returns on three stocks and market index for a period of last 6 years:

| Year | Sheetal Corp. | Alkem Ltd. | Standard Investment | Market Index |
|---|---|---|---|---|
| 1 | 10% | 4% | 3% | 5.0% |
| 2 | 12% | 7% | 4% | 6.0% |
| 3 | 16% | 19% | 8% | 7.5% |
| 4 | 18% | 13% | 11% | 10.2% |
| 5 | 20% | 29% | 12% | 13.0% |
| 6 | 23% | 35% | 15% | 17.0% |

You are **required** to

**a.** Construct a minimum risk portfolio of two stocks, assuming that short selling is not allowed.

**b.** Determine the correlation of the portfolio with the market index if a portfolio of all three stocks with equal proportion is being constructed.

# Solution :

**a.**

| Year | $R_s$ | $R_a$ | $R_{st}$ | $R_m$ | $(R_s - \bar{R}_s)$ (a) | $(R_a - \bar{R}_a)$ (b) | $(R_{st} - \bar{R}_{st})$ (c) | $(R_m - \bar{R}_m)$ (d) |
|---|---|---|---|---|---|---|---|---|
| 1 | 10 | 4 | 3 | 5 | –6.5 | –13.84 | –5.83 | –4.78 |
| 2 | 12 | 7 | 4 | 6 | –4.5 | –10.84 | –4.83 | –3.78 |
| 3 | 16 | 19 | 8 | 7.5 | –0.5 | 1.16 | –0.83 | –2.28 |
| 4 | 18 | 13 | 11 | 10.2 | 1.5 | –4.84 | 2.17 | 0.42 |
| 5 | 20 | 29 | 12 | 13 | 3.5 | 11.16 | 3.17 | 3.22 |
| 6 | 23 | 35 | 15 | 17 | 6.5 | 17.16 | 6.17 | 7.22 |
| Σ | 99 | 107 | 53 | 58.7 | | | | |
| Mean | 16.5 | 17.84 | 8.83 | 9.78 | | | | |

| $(R_s - \bar{R}_s)^2$ | $(R_a - \bar{R}_a)^2$ | $(R_{st} - \bar{R}_{st})^2$ | $(R_m - \bar{R}_m)^2$ | a x d | b x d | c x d |
|---|---|---|---|---|---|---|
| 42.25 | 191.55 | 33.94 | 22.85 | 31.07 | 66.16 | 27.87 |
| 20.25 | 117.51 | 23.33 | 14.29 | 17.01 | 40.98 | 18.30 |
| 0.25 | 1.3456 | 0.69 | 5.198 | 1.14 | –2.645 | 1.892 |
| 2.25 | 23.426 | 4.71 | 0.176 | 0.63 | –2.033 | 0.911 |
| 12.25 | 124.55 | 10.05 | 10.37 | 11.27 | 35.94 | 10.21 |
| 42.25 | 294.47 | 38.07 | 52.13 | 46.93 | 123.9 | 44.55 |
| 119.5 | 752.83 | 110.77 | 105.01 | 108.05 | 262.3 | 103.7 |

$\bar{R}_s = \frac{99}{6} = 16.5$ $\qquad \sum(R_s - \bar{R}_s)^2 = 119.5$ $\qquad \sum(R_s - \bar{R}_s)(R_m - \bar{R}_m) = 108.05$

$\bar{R}_a = \frac{107}{6} = 17.84$ $\qquad \sum(R_a - \bar{R}_a)^2 = 752.83$ $\qquad \sum(R_a - \bar{R}_a)(R_m - \bar{R}_m) = 262$

$\bar{R}_{st} = \frac{53}{6} = 8.84$ $\qquad \sum(R_{st} - \bar{R}_{st})^2 = 110.83$ $\qquad \sum(R_{st} - \bar{R}_{st})(R_m - \bar{R}_m) = 103.7$

$\bar{R}_m = \frac{58.7}{6} = 9.78$ $\qquad \sum(R_s - \bar{R}_s)^2 = 105.01$

$$\sigma_s = \sqrt{\frac{119.5}{6}} = 4.46 \qquad \sigma_m = \sqrt{\frac{105}{6}} = 4.18 \qquad Cov_{sm} = \frac{108}{6} = 18$$

$$\sigma_a = \sqrt{\frac{752.83}{6}} = 11.20 \qquad Cov_{am} = \frac{262.3}{6} = 43.72$$

$$\sigma_{st} = \sqrt{\frac{110.83}{6}} = 4.30 \qquad Cov_{stm} = \frac{103.7}{6} = 17.28$$

Beta of the stocks

$$\beta_s = \frac{18}{17.47} = 1.03$$

$$\beta_a = \frac{43.72}{17.47} = 2.5$$

$$\beta_{st} = \frac{17.28}{17.47} = 0.99$$

Covariance between two stock can be calculated

$$Cov_{xy} = \beta_x \beta_y \sigma_m^2$$

$$Cov_{SA} = \beta_s \beta_a \sigma_m^2$$

= 1.03 × 2.5 × 17.47 = 44.98 $(\%)^2$

$$Cov_{sst} = \beta_s \beta_{st} \sigma_m^2$$

= 1.03 × 0.99 × 17.47 = 17.81$(\%)^2$

$$Cov_{ast} = \beta_a \beta_{st} \sigma_m^2$$

= 2.5 × 0.99 × 17.47 = 43.24$(\%)^2$

Correlation coefficient between stocks

$$\rho_{sa} = \frac{44.98}{4.46 \times 11.20} = 0.9$$

$$\rho_{sst} = \frac{17.81}{4.46 \times 4.30} = 0.93$$

$$\rho_{ast} = \frac{43.24}{11.20 \times 4.30} = 0.9$$

For min$^{m}$ risk portfolio

$$\rho_{AB} < \frac{\sigma_A}{\sigma_B} \text{ where } \sigma_A<\sigma_B$$

$$\rho_{sa} = 0.90 > \frac{4.46}{11.20}$$

$$\rho_{sst} = 0.93 < \frac{4.30}{4.46} = 0.96$$

This satisfies the condition

$$\rho_{ast} = 0.90 > \frac{4.30}{11.20} = 0.38$$

Only combination of Sheetal Corporation and Standard investment is fulfilling the criteria and minimum risk portfolio can be formed using these two stocks.

**b.** Portfolio beta

$$\beta_p = \frac{1.03 + 2.5 + 0.99}{3} = 1.51$$

Correlation of portfolio with respect to market index

$$\beta_p = \rho_p \frac{\sigma_p}{\sigma_m}$$

$$\rho_p = \frac{\beta_p \sigma_m}{\sigma_p} = \frac{1.51 \times 4.58}{\sigma_p}$$

**Portfolio risk**

$$\sigma_p^2 = \left(\frac{1}{3}\right)^2 \times (4.46)^2 + \left(\frac{1}{3}\right)^2 \times (11.20)^2 + \left(\frac{1}{3}\right)^2 \times (4.30)^2 + 2 \times \frac{1}{3} \times \frac{1}{3} \times 44.98 + 2 \times \frac{1}{3} \times \frac{1}{3} \times 17.81 + 2 \times \frac{1}{3} \times \frac{1}{3} \times 43.24$$

$$\sigma_p^2 = 41.76$$

$$\sigma_p = 6.4626$$

**Correlation of the portfolio**

$$\rho_p = \frac{1.51 \times 4.18}{6.4626} = 0.98$$

# PROBLEM – 5

There are four stocks A, B, C and D. The returns on these stocks can be explained using three factors viz. long-term interest rates ($\beta_1$), oil prices ($\beta_2$) and exchange rates ($\beta_3$). The average rate of return and the sensitivity of returns on these stocks to the three factors is given below:

| Stocks | $\beta_1$ | $\beta_2$ | $\beta_3$ | Average Rate of Return (%) |
|---|---|---|---|---|
| A | 0.20 | –0.20 | 0.80 | 9.60 |
| B | 0.70 | 0.10 | 0.60 | 16.03 |
| C | 0 | 0.80 | 0.70 | 14.00 |
| D | 0.60 | –0.20 | 0.70 | 13.35 |
| Risk Premium | ? | 7% | ? | |

**Required:**

**a.** Compute the risk premium paid for each factor assuming a market at equilibrium given that the oil price risk premium is 7%.

**b.** Determine the $\beta_1$ and $\beta_3$ coefficients for the following two portfolios assuming that the portfolio is to be insensitive to the oil prices. Also suggest which portfolio is to be selected:

| Stock | Proportion in Portfolio 1 (%) | Proportion in portfolio 2 (%) |
|---|---|---|
| A | 12.5 | 50 |
| B | 25 | 25 |
| C | 12.5 | 12.5 |
| D | 50 | 12.5 |

**c.** The factor sensitivities for stock D are $\beta_1$=0.32, $\beta_2$ =0 and $\beta_3$ =0.57. The expected return on stock D is 12.35%. Is stock D correctly priced?

(Assume the risk free rate of return as 5%)

# Solution :

**a.** A: 9.60 $= \beta_0 + 0.2\beta_1 - 0.2\beta_2 + 0.8\beta_3$

B: 16.03 $= \beta_0 + 0.7\beta_1 + 0.1\beta_2 + 0.6\beta_3$

C: 14.00 $= \beta_0 + 0.0\beta_1 + 0.8\beta_2 + 0.7\beta_3$

D: 13.35 $= \beta_0 + 0.6\beta_1 - 0.2\beta_2 + 0.7\beta_3$

Given that risk-free rate is 5% and $\beta_2 = 7\%$

$\therefore 14 = 5 + 0 + 0.8 \times 7 + 0.7\ \beta_3$

$\therefore \beta_3 = 4.85\%$

Similarly, $9.6 = 5 + 0.2\ \beta_1 - 0.2 \times 7 + 0.8 \times 4.85$

$\beta_1 = 10.6\%$

**b. Portfolio 1:**

$\beta_1$ Coefficient: $0.125\times0.2+0.25\times0.7+0.125\times0+0.5\times0.6 = 0.50$

$\beta_3$ Coefficient: $0.125\times0.8+0.25\times0.6+0.125\times0.7+0.5\times0.7 = 0.6875$

($\beta_2$ has been taken as zero as the portfolio is to be insensitive to the oil prices)

Rp = $0.125\times9.6+0.25\times16.03+0.125\times14+0.5\times13.35 = 13.63\%$

**Portfolio 2:**

$\beta_1$ Coefficient: $0.5\times0.2+0.25\times0.7+0.125\times0+0.125\times0.6 = 0.35$

$\beta_3$ Coefficient: $0.5\times0.8+0.25\times0.6+0.125\times0.7+0.125\times0.7 = 0.725$

($\beta_2$ has been taken as zero as the portfolio is to be insensitive to the oil prices)

Rp = $0.5\times9.6+0.25\times16.03+0.125\times14+0.125\times13.35 = 12.23\%$

Portfolio 1 is better as it provides a higher return.

**c. Required rate of return on portfolio**

$D(R_D) = \beta_0 + 0.32\ \beta_1 + 0.57\beta_3 = 5 + 0.32\times10.6 + 0.57\times4.85 = 11.16\%$

Since the expected return on stock D is 12.35% which is more than the required rate of return, the stock is undervalued

## PROBLEM – 6

Consider the following prices of the stock of Satyam Computers Ltd and the corresponding value of the Sensex:

| End of Month | Satyam Computers (Rs) | Sensex |
|---|---|---|
| March 2000 | 885.80 | 5001.28 |
| April 2000 | 624.00 | 4657.55 |
| May 2000 | 508.35 | 4433.61 |
| June 2000 | 596.45 | 4748.77 |
| July 2000 | 492.00 | 4279.86 |
| August 2000 | 571.90 | 4477.31 |
| September 2000 | 487.50 | 4085.03 |
| October 2000 | 307.25 | 3711.02 |
| November 2000 | 337.90 | 3997.99 |
| December 2000 | 323.25 | 3972.12 |

You are **required** to calculate:

**a.** The characteristic line for stock of Satyam Computers

**b.** The proportions of systematic risk and unsystematic risk in the total risk of the stock of Satyam Computers

# Solution :

**a. The returns on Satyam Computers and sensex are as follows:**

| Month | Return on the stock of Satyam Computers (y) % | $(Y-\bar{Y})$ | $(Y-\bar{Y})^2$ | Return on Sensex (x)% | $(X-\bar{X})$ | $(X-\bar{X})^2$ | $(Y-\bar{Y})(X-\bar{X})$ |
|---|---|---|---|---|---|---|---|
| April | –29.56 | –20.88 | 435.974 | –6.87 | -4.57 | 20.885 | 95.422 |
| May | –18.53 | –9.85 | 97.023 | –4.81 | -2.51 | 6.300 | 24.724 |
| June | 17.33 | 26.01 | 676.520 | 7.11 | 9.41 | 88.548 | 244.754 |
| July | –17.51 | –8.83 | 77.969 | –9.87 | -7.57 | 57.305 | 66.843 |
| August | 16.24 | 24.92 | 621.006 | 4.61 | 6.91 | 47.748 | 172.197 |
| September | –14.76 | –6.08 | 36.966 | –8.76 | -6.46 | 41.732 | 39.277 |
| October | –36.97 | –28.29 | 800.324 | –9.16 | -6.86 | 47.060 | 194.069 |
| November | 9.98 | 18.66 | 348.196 | 7.73 | 10.03 | 100.601 | 187.160 |
| December | –4.34 | 4.34 | 18.836 | –0.65 | 1.65 | 2.723 | 7.161 |
| | $\Sigma y$ = –78.12, y = –8.68 | | $\Sigma(Y-\bar{Y})^2$ = 3112.814 | $\Sigma x$ = –20.67, x = –2.3 | | $\Sigma(X-\bar{X})^2$ = 412.902 | 1031.607 |

**The regression equation between the two can be determined as follows:**

$$\text{Covariance} = \frac{1031.607}{9} = 114.62(\%)^2$$

$$\text{Variance of Market } (\sigma^2 x) = \frac{412.902}{9} = 45.88(\%)^2$$

$$\text{Variance of Stock} (\sigma^2 y) = \frac{3112.814}{9} = 345.87(\%)^2$$

$$\beta = \frac{\text{Cov}(x,y)}{\sigma^2 x} = \frac{114.62}{45.88} = 2.5$$

$$\alpha = \bar{y} - b \times \bar{x}$$

= -8.68 – b ×(-2.3) = - 8.68 – 2.5 (-2.3) = -2.3

Characteristic line: $R_i$ = -2.93 +2.5 × $R_m$

Where $R_i$ is the return on Satyam Computers and $R_m$ is the return on the market.

**b. Proportion of systematic Risk = $r^2$**

$$r = \frac{Cov(x,y)}{\sigma_x \sigma_y} = \frac{114.62}{6.77 \times 18.6} = 0.91$$

$r^2$ = 0.83

**Proportion of Unsystematic Risk**

= $(1 - r^2)$

= 1 – 0.83 = 0.17

# PROBLEM – 7

Consider the following prices of a stock during January 2000 and January 2001:

| Trading Days | January 2000 | January 2001 |
|---|---|---|
| 1 | 130 | 500 |
| 2 | 133 | 492 |
| 3 | 133 | 472 |
| 4 | 139 | 460 |
| 5 | 139 | 416 |
| 6 | 141 | 392 |
| 7 | 142 | 430 |
| 8 | 141 | 392 |
| 9 | 144 | 416 |

Based on the above prices, test for the weak form of market efficiency using auto-correlation test and comment on the result.

# Solution :

| Trading day | Change in prices | | | | | | |
|---|---|---|---|---|---|---|---|
| | Jan 2000 (x) | Jan 2001 (y) | $(X-\overline{X})$ | $(X-\overline{X})^2$ | $(Y-\overline{Y})$ | $(Y-\overline{Y})^2$ | $(X-\overline{X})(Y-\overline{Y})$ |
| 2 | 3 | −8 | 1.25 | 1.563 | 2.5 | 6.25 | 3.125 |
| 3 | 0 | −20 | -1.75 | 3.063 | -9.5 | 90.25 | 16.625 |
| 4 | 6 | −12 | 4.25 | 18.063 | -1.5 | 2.25 | -6.375 |
| 5 | 0 | −44 | -1.75 | 3.063 | -33.5 | 1122.25 | 58.625 |
| 6 | 2 | −24 | 0.25 | 0.063 | -13.5 | 182.25 | -3.375 |
| 7 | 1 | 38 | -0.75 | 0.563 | 48.5 | 2352.25 | -36.375 |
| 8 | -1 | −38 | -2.75 | 7.563 | -27.5 | 756.25 | 75.625 |
| 9 | 3 | 24 | 1.25 | 1.563 | 34.5 | 1190.25 | 43.125 |
| | $\Sigma x = 14$<br>$\overline{x} = 1.75$ | $\Sigma y = -84$<br>$\overline{y} = -10.5$ | | $\Sigma(X-\overline{X})^2$<br>=35.504 | | $\Sigma(Y-\overline{Y})^2$<br>= 5702 | 151.00 |

$$\text{Covariance(x,y)}= \frac{151}{8} = 18.88(\%)^2$$

$$\sigma^2 x = \frac{35.504}{8} = 4.44(\%)^2$$

$$\sigma^2 y = \frac{5702}{8} = 712.75(\%)^2$$

$$\text{Correlation}(x,y) = \frac{\text{Cov}(x,y)}{\sigma_x \sigma_y}$$

$$= \frac{18.88}{2.11 \times 26.7} = 0.34$$

Hence, the correlation coefficient (r) between the above price changes is 0.34. Therefore, we can say that the test does support the weak form efficiency.

# PROBLEM – 8

Mr. A. Rathi is testing the weak form efficient market hypothesis on the Indian stock market. For this he has collected the data on a leading market index for the last 15 trading days. This is given below:

| Trading day | Market Index |
|---|---|
| 1 | 4500 |
| 2 | 4550 |
| 3 | 4400 |
| 4 | 4350 |
| 5 | 4300 |
| 6 | 4330 |
| 7 | 4400 |
| 8 | 4445 |
| 9 | 4440 |
| 10 | 4370 |
| 11 | 4380 |
| 12 | 4365 |
| 13 | 4500 |
| 14 | 4560 |
| 15 | 4600 |

You are **required** to perform a runs test and determine the independence of data at 10% level of significance.

# Solution :

| Trading Day | Market Index | Price Change |
|---|---|---|
| 1 | 4500 | |
| 2 | 4550 | + |
| 3 | 4400 | – |
| 4 | 4350 | – |
| 5 | 4300 | – |
| 6 | 4330 | + |
| 7 | 4400 | + |
| 8 | 4445 | + |
| 9 | 4440 | – |
| 10 | 4370 | – |
| 11 | 4380 | + |
| 12 | 4365 | – |
| 13 | 4500 | + |
| 14 | 4560 | + |
| 15 | 4600 | + |

r = 7

$$\mu_r = \frac{2n_1 n_2}{n_1 + n_2} + 1 = \frac{2 \times 8 \times 6}{14} + 1 = 7.857$$

$$\sigma_r = \sqrt{\frac{(\mu-1)(\mu-2)}{n_1 + n_2 - 1}} = \sqrt{\frac{(7.857-1)(7.857-2)}{13}} = \sqrt{\frac{6.857 \times 5.857}{13}} = 1.76$$

**where,**

r = Total number of runs

$n_1$ = No. of positive price changes

$n_2$ = No. of negative price changes

At α = 0.10, Z = 1.65

The lower limit: $\mu_r - Z\sigma_r$ = 7.857 – (1.65 x 1.758)

= 7.857 – 2.901 = 4.956

The upper limit : $\mu_r + Z\sigma_r$ = 7.857 + (1.65 x 1.758)

= 7.857 + 2.901 = 10.758

Since the observed number of runs of 7 falls within the lower and upper limits it seems to indicate that the prices are independent at 10% level of significance.

## PROBLEM – 9

The stock research division of M/s Kothari Investment services has developed ex-ante probability distribution for the likely economic scenarios over the next one year and estimates the corresponding one period rates of return on stocks A, B and market index as follows:

| Economic Scenarios | Probability | One period rate of return % | | |
|---|---|---|---|---|
| | | Stock A | Stock B | Market |
| Recession | 0.15 | -15 | -3 | -10 |
| Low growth | 0.25 | 10 | 7 | 13 |
| Medium growth | 0.45 | 25 | 15 | 18 |
| High growth | 0.15 | 40 | 25 | 32 |

The expected risk-free real rate of return and the premium for inflation are 3.0% and 6.5% p.a. respectively.

As an analyst in a research division you are **required** to :

**a.** Calculate the following for stock A and B

**i.** Expected return

**ii.** Covariance of returns with the market returns

**iii.** Beta

**b.** Recommend for fresh investment in any of these two stocks. Show all the necessary calculations.

# Solution :

**a. i. Expected return on stock = E($R_A$)**

$$\sum_{s=1}^{n} R_S P_S$$

$= 0.15\,(-15) + 0.25 \times 10 + 0.45 \times 25 + 0.15 \times 40 = 17.5\%$

E($R_B$) $= 0.15 \times (-3) + 0.25 \times 7 + 0.45 \times 15 + 0.15 \times 25 = 11.8\%$

E($R_M$) $= 0.15 \times (-10) + 0.25 \times 13 + 0.45 \times 18 + 0.15 \times 32 = 14.65\%$

**ii. Covariances**

$$COV_{AM} = \sum_{s=1}^{n} \left[R_{A_s} - E(R_A)\right]\left[R_{M_s} - E(R_M)\right]P_S$$

$$= 0.15\,[(-15) - 17.5]\,[(-10) - 14.65] + 0.25\,[10 - 17.5]\,[13 - 14.65] + 0.45\,[25 - 17.5]\,[18 - 14.65] + 0.15\,[40 - 17.5]\,[32 - 14.65]$$

$$= 193.13\,(\%)^2$$

$$COV_{BM} = 0.15\,[(-3) - 11.8]\,[(-10) - 14.65] + 0.25\,[7 - 11.8]\,[13 - 14.65] + 0.45\,[15 - 11.8]\,[18 - 14.65] + 0.15\,[25 - 11.8]\,[32 - 14.65]$$

$$= 95.88(\%)^2$$

$$VAR_M\,(\sigma_m^2) = 0.15\,[(-10) - 14.65]^2 + 0.25\,[13 - 14.65]^2 + 0.45\,[18 - 14.65]^2 + 0.15\,[32 - 14.65]^2$$

$$= 142.03(\%)^2$$

**iii.**

$$\beta_A = \frac{COV_{AM}}{\sigma_M^2} = \frac{193.13}{142.03} = 1.36$$

$$\beta_B = \frac{COV_{BM}}{\sigma_M^2} = \frac{95.88}{142.03} = 0.675 \approx 0.68$$

**b.** For ex-ante SML $R(r_i) = r_0 + r_i\beta_{im}$

Where,

$r_0$ = Intercept of SML

$r_i$ = Slope of the SML

If the assumptions at the CAPM are correct, then

$R(r_i) = r_f + [E(r_m) - r_f]\,\beta_{im}$

Where, $r_f$ = Risk free rate

$E(r_m) - r_f$ = Slope of SML

Given $r_f = 3.0 + 6.5 = 9.5\%$

Where, $r_f$ = Inflation adjusted nominal risk free rate.

**i.** $R(r_A) = 9.5 + 1.36 \times [14.65 - 9.5] = 16.50\%$

$\alpha_A = E(R_A) - R(r_A) = 17.50 - 16.50 = 1.0\%$

Hence, A is under priced.

**ii.** $R(r_B) = 9.5 + 0.675 \times [14.65 - 9.5] = 12.98\%$

$\alpha_B = E(R_B) - R(r_B) = 11.80 - 12.98 = -1.18\%$

Hence, B is over priced.

Therefore, it is recommended to invest in Stock A.

# PROBLEM – 10

Consider the following data for two companies and the market:

| Company/Market | Beta | Standard Deviation (%) | Covariance with Sensex ($\%^2$) |
|---|---|---|---|
| Zee Telefilms | N.A. | 45 | 205 |
| Padmalay Telefilms | 1.2 | 40 | N.A. |
| Sensex | 1.0 | 15 | 225 |

Further it is gathered that risk free interest is 7%. Considering the assumptions of regression (Characteristic) line hold good you are **required** to find

**a.** i. Beta of Zee Telefilms

ii. Covariance of return on Padmalay Telefilms with that of return on sensex

**b.** The coefficients of correlation between

i. Return on Zee Telefilms and return on sensex

ii. Return on Padmalaya Telefilms and return on sensex

**c.** The variance of the portfolio formed using Zee Telefilms and Padmalaya Telefilms in the proportion of 2/3 and 1/3 respectively.

**d.** Whether the unsystematic risk of the portfolio is less than individual companies?

(β of portfolio is weighted average betas of underlying stocks).

# Solution :

a. i. $\beta_{zee} = \dfrac{\text{Cov (Zee, M)}}{\text{Var (M)}} = \dfrac{0.0205}{(0.0225)} = 0.91$

ii. $\text{Cov (Pad, M)} = \beta_{Pad} \times \text{Var (M)} = 1.2 \times 0.0225$

$= 0.027 \text{ i.e., } 270\ (\%^2)$

b. i. $\rho_{zee, M} = \dfrac{\text{Cov(zee, M)}}{\sigma_{zee} \times \sigma_M} = \dfrac{0.0205}{0.45 \times \sqrt{0.0225}} = 0.304$

ii. $\rho_{Pad, M} = \dfrac{\text{Cov(Pad, M)}}{\sigma_{Pad} \times \sigma_M} = \dfrac{0.027}{0.40 \times \sqrt{0.0225}} = 0.45$

c. $\text{Var (Portfolio)} = W_{zee}^2\ \sigma_{zee}^2 + W_{Pad}^2\ \sigma_{Pad}^2 + 2W_{zee}, W_{Pad}\ \text{Cov (Zee, Pad)}$

$W_{zee} = \dfrac{2}{3};\ \sigma_{zee} = 0.45$

$W_{Pad} = \dfrac{1}{3};\ \sigma_{Pad} = 0.40$

From the assumptions of characteristic (Regression) line we get

$\text{Cov (zee, Pad)} = \beta_{zee} \times \beta_{Pad} \times \text{Var (M)}$

$= 0.91 \times 1.2 \times 0.0225 = 0.025 \text{ i.e. } 250(\%^2)$

$\text{Variance (Portfolio)} = \left(\dfrac{2}{3}\right)^2 \times (0.45)^2 + \left(\dfrac{1}{3}\right)^2 \times (0.40)^2 + 2 \times \dfrac{2}{3} \times \dfrac{1}{3} \times 0.025$

$= 0.119 \text{ i.e., } 1190\ (\%^2)$

d. Unsystematic Risk of Zee Telefilms $= \left(1 - \rho_{zee, M}^2\right) \sigma_{zee}^2$

$= [1 - (0.304)^2] \times (0.45)^2$

$= 0.184 \text{ i.e., } 1840\ (\%^2)$

Unsystematic Risk of Padmalay Telefilms $= \left(1 - \rho_{Pad, M}^2\right) \sigma_{Pad}^2$

$= [1 - (0.45)^2] \times (0.40)^2$

$= 0.128 \text{ i.e., } 1280\ (\%^2)$

$\beta_{Portfolio} = \dfrac{2}{3} \beta_{zee} + \dfrac{1}{3} \beta_{Pad} = \dfrac{2}{3} \times 0.91 + \dfrac{1}{3} \times 1.2 = 1.007$

$$\rho_{Port,M} = \frac{Cov(Port, M)}{\sigma_{Port} \times \sigma_M} = \beta_{Port} \times \frac{\sigma_M}{\sigma_{Port}}$$

$$= 1.007 \times \sqrt{\frac{0.0225}{0.119}}$$

$$= 0.438$$

$$\text{Unsystematic risk of portfolio} = \left(1 - \rho^2_{Port,M}\right)\sigma^2_{Port}$$

$$= [1 - (0.438)^2] \times 0.119$$

$$= 0.096 \text{ i.e., } 960\ (\%^2)$$

Therefore, we find that the unsystematic risk of the portfolio is less than that of individual stocks. From the result it can be implied that because of constitution of portfolio unsystematic return reduces.