Lecture Notes 6

Scenario Analysis and Application of Joint Probability

Question

A&R Ltd. has under its consideration a project with an initial investment of ₹ 90,00,000. Three probable cash inflow scenarios with their probabilities of occurrence have been estimated as below:

Annual cash inflow (₹)	20,00,000	30,00,000	40,00,000
Probability	0.2	0.7	0.1

The project life is 5 years and the desired rate of return is 18%. The estimated terminal values for the project assets under the three probability alternatives, respectively, are ₹ 0, ₹ 20,00,000 and ₹ 30,00,000.

You are required to:

i. **Calculate** the probable NPV;

ii. **Calculate** the worst-case NPV and the best-case NPV; and

iii. **State** the probability occurrence of the worst case, if the cash flows are perfectly positively correlated over time.

Answer :

i. **Calculation of Net Present Value (NPV)**

Year	Prob. = 0.2		Prob. = 0.7		Prob. = 0.1				
	Cash flow	Probable cash flow	Cash flow	Probable cash flow	Cash flow	Probable cash flow	Total Cash flow	PVF@ 18%	PV of Total cash flow
0							(90,00,000)	1.000	(90,00,000)
1	20,00,000	4,00,000	30,00,000	21,00,000	40,00,000	4,00,000	29,00,000	0.847	24,56,300
2	20,00,000	4,00,000	30,00,000	21,00,000	40,00,000	4,00,000	29,00,000	0.718	20,82,200
3	20,00,000	4,00,000	30,00,000	21,00,000	40,00,000	4,00,000	29,00,000	0.608	17,63,200
4	20,00,000	4,00,000	30,00,000	21,00,000	40,00,000	4,00,000	29,00,000	0.515	14,93,500
5	20,00,000	4,00,000	30,00,000	21,00,000	40,00,000	4,00,000	29,00,000	0.437	12,67,300
5	0	0	20,00,000	14,00,000	30,00,000	3,00,000	17,00,000	0.437	7,42,900
Net Present Value (NPV)									**8,05,400**

ii. **Worst and Best case is the case where expected annual cash inflows are minimum and maximum respectively.**

Calculation of Worst Case and Best Case NPV:

Year	PVF@ 18%	Worst case		Best Case	
		Cash flows	PV of Cash flows	Cash flows	PV of Cash flows
0	1.000	(90,00,000)	(90,00,000)	(90,00,000)	(90,00,000)
1	0.847	20,00,000	16,94,000	40,00,000	33,88,000
2	0.718	20,00,000	14,36,000	40,00,000	28,72,000
3	0.608	20,00,000	12,16,000	40,00,000	24,32,000
4	0.515	20,00,000	10,30,000	40,00,000	20,60,000
5	0.437	20,00,000	8,74,000	40,00,000	17,48,000
5	0.437	0	0	30,00,000	13,11,000
NPV			**(27,50,000)**		**48,11,000**

Worst case NPV = ₹ (27,50,000)

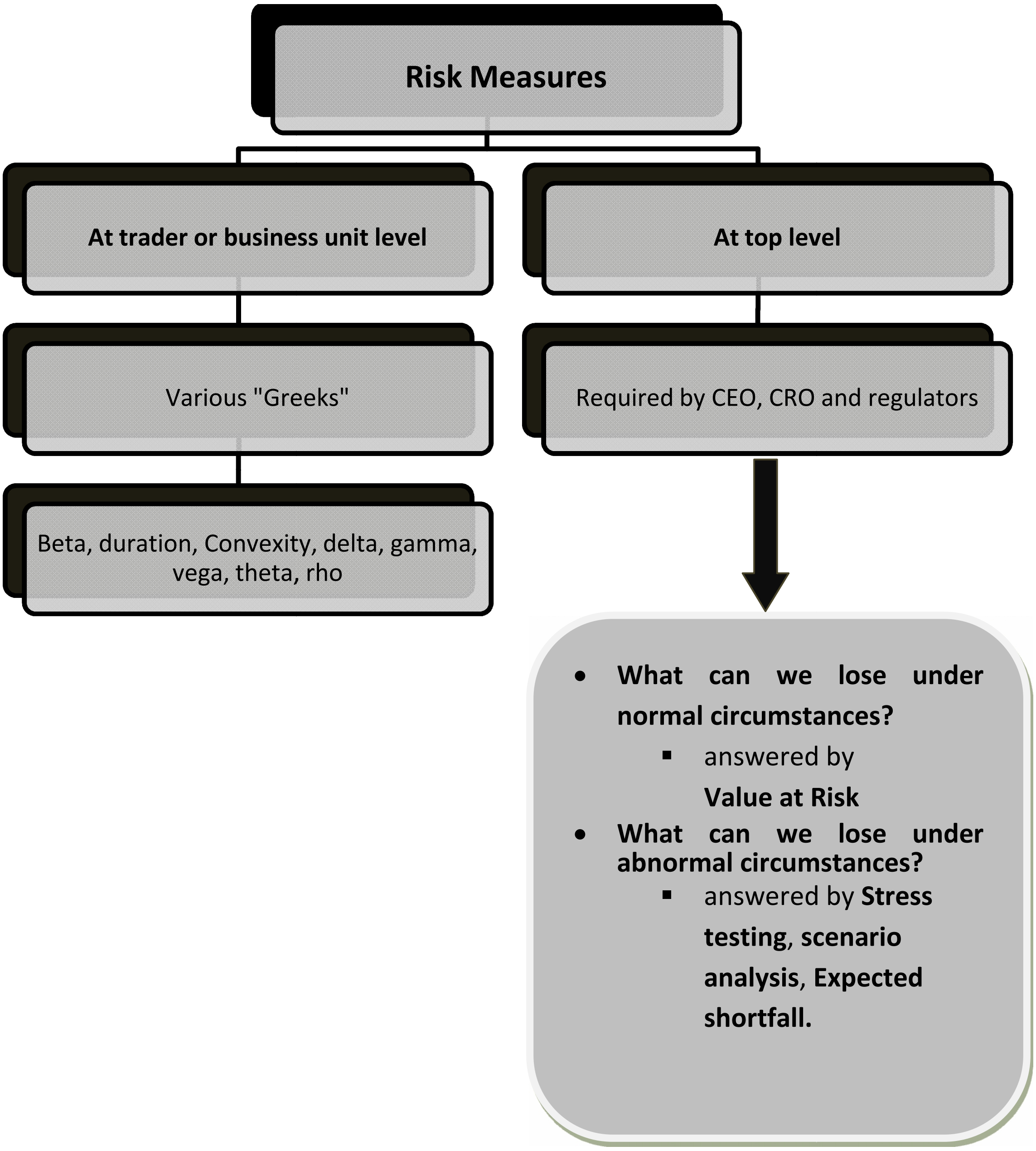
Best Case NPV = ₹ 48,11,000

iii. The cash flows are perfectly positively correlated over time means cash flow in first year will be cash flows in subsequent years. The cash flow of ₹20,00,000 is the worst case cash flow and its probability is 20%, thus, possibility of worst case is 20% or 0.2.

Lecture Notes 6

Chapter 5 : Risk Model

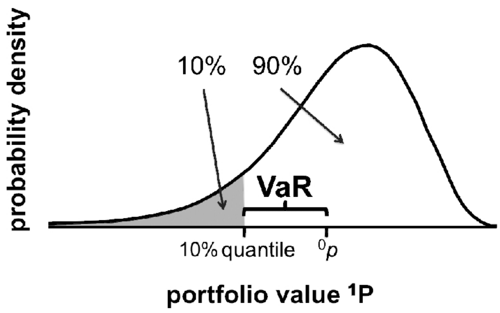
1. Risk Measures



2. Interpretation of VaR

- Amount at stake
- If 5% daily VaR is ₹ 40 cores, it can be interpreted in 2 ways
  - we are 95% confident that the maximum possible loss would be 40 crores, with a 5% chance of being exceeded.
  - In 5% of the worst circumstances, loss will be at least 40 crores.

Note - VaR is not Expected loss – That means, it is not a measure of Mean – it is a quantile.



3. VaR Parameters

- **Confidence level : i.e., probability**
  - obviously a 99% VaR is bigger, i.e., more conservative than 95% VaR.

BASEL norms consider 99% VaR for Market Risk Charge and 99.9% VaR for Credit Risk Charge.

- **Time Period**
  - Obviously 10 day VaR is bigger than 1 day VaR
  - BASEL norms consider 10 day VaR for Market Risk Charge and 1 Year VaR for Credit Risk Charge.
  - *We use the square root rule for VaR conversion. This means, n day VaR = 1 day VaR $\times$ SQRT(n). We consider 250 trading days in a year. So, daily VaR = Annual VaR/SQRT(250).*

Example

Annual VaR = ₹ 2,40,000

$\therefore$ daily VaR = $2{,}40{,}000 \times \frac{1}{\sqrt{250}} =$ ₹15,178

Example

Daily VaR = 10,000

$\therefore$ 10 days VaR = $10{,}000 \times \sqrt{10} =$ ₹31,622,78

Example

Two months VaR will exceed one-month VaR by ………..%.

- 2 Month VaR = 1 month VaR × SQRT(2)
  = 1 month VaR × 1.4142

Hence, 2 month VaR exceeds 1 month VaR by 41.42%.

Example

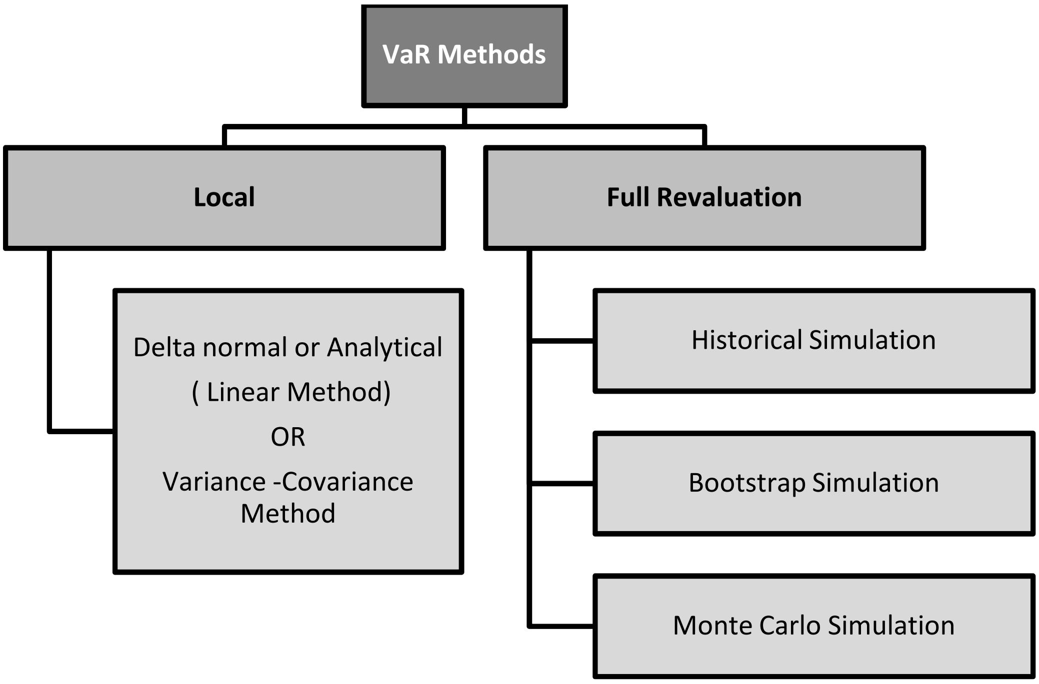
If 10 days VaR = 50,000, calculate 50 day VaR

➢ 50 day VaR = $50{,}000 \times \sqrt{5} =$ ₹1,11,803

Note :

- ✓ ***We can only apply the square root rule when ($\mu = 0$)***
- ✓ ***We use Shorter time periods in case of model validation and Back Testing.***
- ✓ ***In fact time period to be used for VaR depends upon <u>liquidity</u> of the financial market in which the institution operates - How quickly will one be able to liquidate a position.***

4. VaR Methods



5. Delta Normal Method

Warm - Up

Question

An analyst would like to know the VAR for a portfolio consisting of two asset classes: long-term government bonds issued in the United States and long-term government bonds issued in the United Kingdom. The expected monthly return on US bonds is 0.85 percent, and the standard deviation is 3.20 percent. The expected monthly return on UK bonds, in US dollars, is 0.95 percent, and the standard deviation is 5.26 percent. The correlation between the US dollar returns of UK and US bonds is 0.35. The portfolio market value is $100 million and is equally weighted between the two asset classes. Using the analytical or variance–covariance method, compute the following:

A. 5 percent monthly VAR.
B. 1 percent monthly VAR.
C. 5 percent weekly VAR.
D. 1 percent weekly VAR.

Answer :

$$E(R_p) = up(monthly) = \frac{0.85+0.95}{2} = 0.9\%$$

$$\sigma_p = \sqrt{(.5)^2(3.2)^2 + (0.5)^2(5.26)^2 + 2(.5)(.5)(3.2)(5.26)(.35)}$$

$$= \sqrt{2.56+6.9169+2.9456} = \mathbf{3.52\%}$$

VaR(5%) monthly = $(\mu - z\sigma)$% of 100mn

[0.9 - (1.65 × 3.52)]% of 100 = $ 4.908mn

VaR (1%)monthly= (0.9 - (2.33 × 3.52) % of 100 = $ 7.302mn

(5%) Weekly VaR = $\left[\frac{0.9}{4} - \left(1.65 \times \frac{3.52}{\sqrt{4}}\right)\right]$% of 100mn = $2.679mn

(1%) weekly VaR = $\left[\frac{0.9}{4} - \frac{(2.33 \times 3.52)}{\sqrt{4}}\right]$% of 100mn = $3.8758mn

Theory of Delta-normal Approach

- Assumptions :
  - Normal distribution
  - linear relation
- This is parametric approach i.e., this approach **assumes** normal distribution and required two parameters i.e., μ and σ
- We should remember the following z values

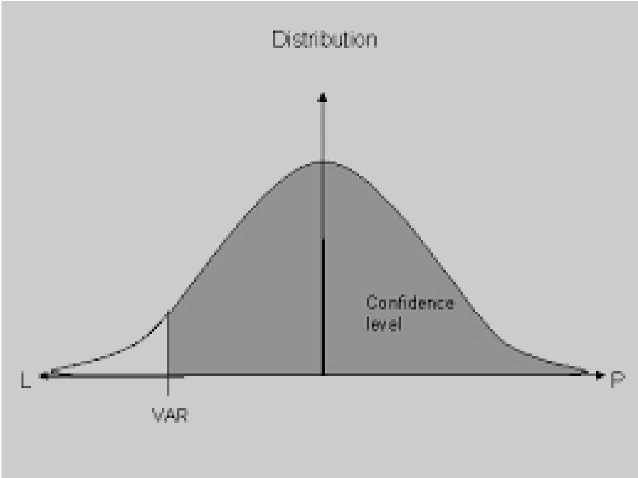
CONFIDENCE LEVEL	AREA IN THE LEFT TAIL (LOSS REGION)	Z - VALUE
99%	1%	2.33
95%	5%	1.65
90%	10%	1.28
84%	16%	1

In case some other confidence level is asked in the exam, you need to look at the normal distribution table :

TABLE C

Normal probability distribution table

NUMBER OF STANDARD DEVIATIONS FROM MEAN (Z)	AREA TO THE LEFT OR RIGHT (ONE TAIL)	NUMBER OF STANDARD DEVIATIONS FROM MEAN (Z)	AREA TO THE LEFT OR RIGHT (ONE TAIL)
0.00	0.5000	1.55	0.0606
0.05	0.4801	1.60	0.0548
0.10	0.4602	1.65	0.0495
0.15	0.4404	1.70	0.0446
0.20	0.4207	1.75	0.0401
0.25	0.4013	1.80	0.0359
0.30	0.3821	1.85	0.0322
0.35	0.3632	1.90	0.0287
0.40	0.3446	1.95	0.0256
0.45	0.3264	2.00	0.0228
0.50	0.3085	2.05	0.0202
0.55	0.2912	2.10	0.0179
0.60	0.2743	2.15	0.0158
0.65	0.2578	2.20	0.0139
0.70	0.2420	2.25	0.0122
0.75	0.2264	2.30	0.0107
0.80	0.2119	2.35	0.0094
0.85	0.1977	2.40	0.0082
0.90	0.1841	2.45	0.0071
0.85	0.1711	2.50	0.0062
1.00	0.1557	2.55	0.0054
1.05	0.1469	2.60	0.0047
1.10	0.1357	2.65	0.0040
1.15	0.1251	2.70	0.0035
1.20	0.1151	2.75	0.0030
1.25	0.1056	2.80	0.0026
1.30	0.0986	2.85	0.0022
1.35	0.0885	2.90	0.0019
1.40	0.0808	2.95	0.0016
1.45	0.0735	3.00	0.0013
1.50	0.0668		



Why do we call this method linear?

This method is used when portfolio positions are **Linear function** of the risk factors.

$$dp = k \times dr$$

Where,

dp = change in portfolio value
dr = change in risk factor
k = measure of sensitivity

Position	Risk factor	Sensitivity Measure
➢ Stock	Market Index	Beta
➢ Bond	Yield	Duration
➢ Forward, Futures, and Swap	Underlying Asset	Delta
➢ Option*	Underlying Asset	Delta

**Options are non-linear derivative - their delta keeps on changing. So, we should not use Delta-normal method for an optioned portfolio.*

Thus, it is recommended that the Delta normal should **NOT** be used for options, we may use Delta-gamma method or better still, we should use the Full Revaluation Method.

There are certain 'misbehaved' position such as Bonds with embedded options like - Callable Bond, Puttable Bond, Mortgage Backed Securities (MBS).

Even **Delta-Gamma method** should not be used for MBS - we should use **Full Revaluation Method**

Example		
Share Price	=	430
Call Price	=	26
Call Delta	=	0.6
No. of Call Option	=	1000
S.D. of stock	=	2.5%

Calculate VaR of the 1000 call option via delta normal method and at 95% confidence.

Answer :

1000 call is equivalent to (0.6 × 1000) = 600 shares.

It means we have to calculate VaR of 600 shares

At 95%, z = 1.65, σ = 2.5%, V_p = 600 × 430 = ₹ 2,58,000

∴ VaR = (z ×σ) % of V_P

= (1.65 × 2.5)% of 2,50,000 = ₹10,642.50

Comment : *According to delta normal method, this VaR is applicable for 1000 C^+ as well as 1000 C^-. This is ridiculous and unacceptable. If we have 100 C^+, our VaR should be less than ₹ 10,642. If we have 1000 C^- our VaR is more than ₹ 10,642. The gamma adjustment does improve the same, but we don't have it in the curriculum.*

Example

Consider a portfolio with long options (C^+ and P^+). If you measure VaR of this portfolio with Delta-normal method-

Question 1 :

Would the VaR measured by you be correct or understated or overstated?

Answer :

Option buying means Positive Skewness while delta normal method assumes normal distribution, so, VaR is overstated.

Question 2 :

What do you advice to correct for this bias?

Answer :

We may use Delta-gamma method - Options are non-linear exposures - But well behaved (curvature does not change)

Note *- Although Delta-gamma method will improve our answer, but it will not give the accurate answer - doesn't matter because -* ***"Risk Management is an art of approximation"***

Question 3:

Can we use delta-gamma method for calculating VaR of Callable bond, puttable bond or MBS?

Answer :

No, Because they are "misbehaved"

Question 4 :

Summarized the discussion of Delta normal method

Answer :

- This method of calculating VaR can be used when the risk factor follows **normal**

Distribution and the portfolio is **linearly related** to the Risk factor.

- If portfolio is **not linear** but well behaved (e.g., straight bonds & options) we may use **Delta-gamma method.**
- If portfolio is **non-linear** and **misbehaved**, we should use **Full Revaluation approach.**