Lecture Note 8

Chapter 5 : Risk Model

7. Coherent Risk Measures

A risk measure is said to be "coherent" if it satisfies all the following 4 properties :

a. **Sub-additivity** : It means risk of a combination or portfolio should be less than or equal to the sum total of risk of individual components.

$$\rho(x+y) \leq \rho(x) + \rho(y)$$

Note : VaR often does not satisfy this property when the distribution is super fat.

hint : Loan 1 : ρ D = 2%,LGD = 100% = 10 crore

Loan 2 : ρ D = 2%,LGD = 100% = 10 crore

If both these loans are independent and identical (IID) and they are merged together, the probability distribution would be

Loan Amount	Probability	Cumulative Probability
-20 cr	.02 × .02 = 0.04%	0.04%
-10 cr	(.98 ×.02) + (.98 × .02) = 3.92%	3.96%
0	(.98 × .98) = 96.04%	100%

Hence the 97.5% VaR is 10 crores

This means that the VaR of the merged loan portfolio is greater than the Sum total of individual VaRs.

So what?

The problem is that bank credit mangers would not like to hold a diversified loan portfolio when they find that VaR of their portfolio is actually higher.

So, VaR may discourage diversification.

Limitations of VaR

1. It is uninformative of tail losses
2. It can create perverse incentive structure
3. It can discourage diversification
4. It is NOT sub-additive

b. **Homogeneity :** If you double your position, risk method should get doubled.

$$\rho(kx) = k \times \rho(x)$$

c. Monotonicity : If position 'x' always yields a value or return greater than position 'y'' risk of position 'x' should be loss than that of 'y'.

Thus if x > y, for odd out outsources, than :

$$\rho(x) < \rho(y)$$

d. Risk free condition : If we add cash, i.e. risk free asset to a risky position, our risk measure reduces by the amount of cash (k)

$$\rho(x + \mu) = \rho(x) - k$$

Comment : VaR satisfies the last 3 properties but often does NOT satisfy the first one. So, VaR is not a coherent risk measure.

8. Expected shortfall or Conditional VaR

Imagine a trading firm which has imposed a 1%daily VaR limit of ₹ 5 cores for a trader. This means the trader should not hold positions whose 1% daily VaR exceeds ₹ 5 crores.

Now, suppose the trader has to choose between 2 mutually exclusive positions "x", &, "y". Coincidentally both 'x' & 'y' have 1% daily VaR of ₹ 4.9 crores each. This seems to be perfect.

Now, the expected return of "x" is slightly greater than that of "y"

So, the trader would have the incentive to go for "z". However, it is later known that if actual loss exceeds VaR of 5 crores (we mean to say that the 1% tail event has happened), the expected, i.e., average loss in position "x" is 15 crores while that in position "y" is 6 crores.

$$E\left(\frac{L_x}{(L_x > VaR_x)}\right) = 15 \text{ crores}$$

$$E\left(\frac{Ly}{Ly > VaR_y}\right) = 6 \text{ crores}$$

Ideally, the trader should enter into position "y". However, trader has no incentive to go for "y" as $E(R_x) > E(R_y)$ and the trader's risk-adjusted performance appraisal in this organization shall be undertaken using VaR. Hence, we criticize VaR on following grounds.

Limitations of VaR

1. It is uninformative of tail losses **(remember : risk lies in the tails)**
2. It discourages diversification
3. It is NOT sub-additive
4. It can create perverse incentive Structures
5. It fails to capture liquidity risk in crisis
6. It only focuses on losses and not on gains

"Expected Shortfall"is defined as the expected loss when VaR is expected. In other words, it is the average of the tail losses. Expected shortfall is **BETTER** measure than VaR because :

- informative of tail losses
- encourage diversification
- more stable than VaR
- satisfies sub-additivity property.
- does not create perverse incentives structures.