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Quantitative Methods
- → TVM & DCF
- → Basic Statistics
- → Advanced Statistics
- → Technical Analysis.

TIME VALUE OF MONEY

Los a : Interest rate interpretation

1) Interest rate is the price of money. It can be interpretted in 3 ways —
→ Required rate of return
→ Discount rate
→ Oppurtunity cost

2) • Given the risk involved and other factors, I will make this investment only if I get a return of atleast 14%. → Required Rate

• If I need 40000 1 year from now, the amount I need to invest today $= \frac{40000}{1.14} = 35087.7$
14 → Discount rate

• I can earn 14% elsewhere, so I need 14% from this investment → Oppurtunity cost

los b : components of interest rate

- Risk free Real Rate ①
- Inflation Premium ②
- Risk Premium
  - ③ → Default Risk Premium
  - ④ → Liquidity Risk Premium
  - ⑤ → Maturity Risk Premium

1. RFreal → Every investment involves sacrifice of current consumption. RFreal is the oppurtunity cost of sacrificing current consumption.

2. Inflation Premium → It is the extra return demanded for the expected loss of purchasing power due to inflation.

Note: If an investment provides inflation protection, the investor is not exposed to purchasing power risk. Hence, inflation premium is not required.
Eg: Treasury inflation protected security (TIPS)

Note 2: Inflation protection is a speciality which has to be mentioned in the name of the bond. If nothing is mentioned, there is no ~~p~~ inflation protection. Hence you will demand nominal rate (i.e including inflation premium.

3. Default Risk Premium – Default may be defined as counter party failure in fulfilling its obligations. Treasury securities and certain agency securities guarenteed by the government (Eg. municipal bonds) cannot default. All other securities are exposed to different levels of default risk indicated by their credit rating. Lower the rating, I have probability of default and therefore higher default risk premium demanded by the investor.

Note: 2 popular rating agencies in the world

S&P – Standards & Poor ~~Moodys~~	Moody's	
AAA	Aaa	Investment Grade
AA	Aa	
A	A	
BBB	Baa	
BB	Ba	Junk or Speculative or High Yielding Bond
B	B	
CCC	Caa	
CC	Ca	

4. Liquidity Risk Premium – It refers to the extra return demanded by an investor for the lack of liquidity. Liquidity refers to the ability to create or offload positions with speed and at a fair price. Real Estate for eg. is one of the least liquid assets. Treasuries on the other hand are most liquid.

5. Maturity Risk Premium – The price of a bond maybe given by —

$$P_0 = \frac{FV}{(1+r)^n}$$

→ Discount rate which keeps on changing and accordingly P_0 changes in the opposite Direction.

Since $(1+r)$ is raised to the power n, here comes the maturity effect. Long term investments (n is big) are more volatile than short term investments. This means if r changes, the price of long term bonds changes by a greater percentage than the price of short term bonds. Hence, we required an extra return when we invest for the long term. This extra return is maturity risk premium.

Example Yield on T-Bills (→ Short term) = 6%
Yield on T-Bonds (→ long term) = 7.5%
Calculate the relevant risk premium and name it

→ T-Bills are short term while T-Bonds are long term — both are liquid and both are default free. So, the difference in yield i.e 1.5% represent maturity risk premium

Example Yield on TIPS = 3%
Yield on T-Bills – 5.2%
What does the difference represent?
→ Inflation Premium

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Example Yield on Agency security guaranteed by government - 6%
Yield on Treasury securities = 5.1%.
Assuming both are of the same maturity, what does the difference represent
→ Liquidity Risk Premium

Example Real rate = 8%. Inflation Premium = 3%.
Calculate Nominal Rate
→ Method 1
Nominal rate ≅ Real + Inflation
$= 8 + 3 = 11\%$ (approx)

Method 2
Nominal Rate $= [(1 + \text{real rate})(1 + \text{inflation})] - 1$
$= (1.08 \times 1.03) - 1$
$= 11.24\%$

Example. Risk free rate = 7%. Risk adjusted rate = 12%.
Calculate Risk Premium
→ Method 1
Risk Premium $= 12 - 7 = 5\%$ (approx)
Method 2
Risk Premium $= \frac{1.12}{1.07} - 1 = 4.67\%$

Note: The various components of interest rate can either be incorporated in an additive model (approx) or multiplicative model (exact). We normally use additive model. The tone of the multiplicative model is
[Factor × Factor] − 1

Example Risk Adjusted Nominal rate = 13%.
Inflation Premium = 4%.
Risk Premium = 5%.
Calculate Risk free real rate
→ Method 1
Risk free real rate $= 13 - 4 - 5 = 4\%$.

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Method 2

$$\frac{1.13}{1.04 \times 1.05} - 1 = 3.479\% \text{ (approx)}$$

Pg-24 - Q1 - Q5
Pg-20 - Q1 - Q2

Example Consider a project which is risky. It involves the production of a commodity. The current price of the commodity is 40. We expect 5% inflation. We expect to produce and sell 1000 units at the end of Year 1.
Risk adjusted real rate of return = 11%
Calculate the amount that we're ready to invest today

→ $PV = \frac{FV}{1+r}$

The rule is that nominal cashflows must be discounted at the nominal rate while real cashflows must be discounted at the real rate.

Method I
Nominal "CF" & Nominal "r"
Nominal CF = 1000 × 40 × 1.05
= 42000
Nominal r = (1.11 × 1.05) − 1
= 16.55%

$\therefore PV = \frac{42000}{1.1655} = 36036$

Method II
Real "CF" & Real "r"
Real CF_1 = 1000 × 40 = 40000 → in terms of today's purchasing power

Real r = 11%

$\therefore PV = \frac{40000}{1.11} = 36036$

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6. Calculator Stuff

(a) Decimal Places

2nd — Format $\boxed{4}$ Enter CE|C

(b) Storage and Recall

$$\frac{40000}{1.15} + \frac{70000}{1.17}$$

$\Downarrow$ $\qquad\qquad$ $\downarrow$

STO 1 $\qquad$ STO 2

RCL 1 + RCL 2

Example $\quad \dfrac{70000}{(1.07)^{10}} + \dfrac{90000}{(1.08)^{20}}$

(c) TVM Mode

$\boxed{N}$ $\quad$ $\boxed{I/Y}$ $\quad$ $\boxed{PV}$ $\quad$ $\boxed{PMT}$ $\quad$ $\boxed{FV}$

Interest rate(%) $\qquad$ Annuity

opposite sign

To clear Previous values — 2nd CLR TVM

Example : There is an investment which will pay the following amounts and are to be discounted at different rates because of different levels of risk

Yrs	CF	From whom	disc. rate
4	50000	BB	12.5%
7.5	200000	A	11%
30	1000000	B	13.75%

Calculate the amount to be invested today

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d) BODMAS
2nd Format
2nd Enter ↑
2nd Enter AOS

Example Consider the following portfolio

Stocks	Weight	Expected Return
A	0.5	19 %
B	0.3	25 %
C	0.2	10 %

Calculate expected return of the portfolio

$E(R_p)$ = weighted average
$= 0.5 \times 19 + 0.3 \times 25 + 0.2 \times 10$

LOS C : Stated Annual rate and effective annual rate

1) We know that compound interest involves applying interest on interest. Frequency of compounding refers to the no. of times this is done in a year — let us represent it by m.

- Thus compounded semi-annually implies $m = 2$
- Thus compounded monthly implies $m = 12$
- Thus compounded every 5 month implies $m = \frac{12}{5} = 2.4$
- Thus " daily implies $m = 365$

2) It is a general convention to quote interest rate at per annum. There can be 2 types of interest rate —

- Stated Annual Rate (SAR) - This rate is per annum compounded m times a year — stated means that compounding has not yet been done.
- Effective Annual Rate [EAR] — This rate is per annum compounded annually.

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3) Conversion from SAR to EAR [Answer will be higher

Step 1: Divide SAR by m

Step 2: What you get i.e. $\frac{SAR}{m}$ is the effective periodic rate (EPR)

Step 3: Make a factor of EPR and raise it to the power m i.e. $EAR = (1+EPR)^m - 1$

Alternate Method via TVM mode

Step 1: Same

Step 2: let us calculate the FV of 100 after 1 year

100 ± PV

$\frac{SAR}{m}$ i.e EPR → I/Y

m → N

CPT → FV = [100 + something]

∴ EAR = FV − 100

Examples ① Calculate EAR

case 1 → SAR = 9% per annum compounded monthly
EAR = 9.3807 p.a. compounded annually

case 2 → SAR = 15% p.a. compounded every 4 months
EAR = ~~0~~15.76%

case 3 → SAR = 18% p.a. compounded daily
EAR = 19.71%

4) Conversion from EAR to SAR [Answer will be lower

Method 1

Step 1: Make a factor of EAR i.e (1+EAR)

Step 2: $EPR = [(1+EAR)^{1/m} - 1] \times 100$

Step 3: ~~EPR~~ SAR = EPR × m

16/1 Every 3 months

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Method 2 (Using TVM mode)

Step 1: Understand that a PV of 100 is becoming an FV of 100+EAR in 'm' periods

Step 2: 100 ± PV

100 + EAR → FV

M → N

CPT → I/Y → EPR

Step 3: SAR = EPR x m

Pg 20 Q 4-6

(5) One type of SAR to another type of SAR

Example (1) SAR = 15% m = 2

calculate SAR m = 4

→ Answer will be less than 15

Method 1

SAR (m=2) → EAR → SAR (m=4)

$\frac{15}{2} = 7.5\%$ → $(1.075)^2 - 1$

100 ± PV

7.5 I/Y

2 $\boxed{N}$

CPT FV

− 100

Ex (2) SAR = 12% m = 12

Calculate SAR = I/Y with m = 365

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6) Continuous Compounding

$m \longrightarrow \infty$

Financial variables such as stock price, interest rate, exchange rate, etc. change everymoment and hence the concept of continuous compounding i.e. compounding every moment. By a result in calculus, the future value of \$1 at a rate of i_c which is continuously compounded is given by e^{i_c}.

Example ① If 1000 is deposited at 10% p.a. compounded continuously for 5 years, what is the future value

$\rightarrow$ $FV = 1000 \times (e^{0.1})^5$

$= 1000 \times e^{0.5}$

$= 1648.72$

However usually student like to convert i_c into Effective Rate (i_e) and then use the TVM mode

The conversion formulae's are

$$i_c \underset{\ln(1+i_e)}{\overset{e^{i_c}-1}{\rightleftarrows}} i_e$$

In the above sum $i_e = e^{0.1} - 1$

0.1 2nd e^x =

-1

$\times 100$

$= 10.51$

$\therefore$ 1000 ± PV 10.51 I/Y

5 N CPT FV

Let us fill in the blanks below to practice conversion

Sl. No.	i_c	i_e
1.	14%	$= e^{0.14} - 1 = 15.027380$
2.	$\ln(1.17)$ 15.700375	17%
3.	$\ln(1.04)$ 3.922071	4%
4.	$\ln(.83)$ −18.63	−17%

$10\,\% = e^{0.1} - 1$

$\ln(10+1) = 0.1$

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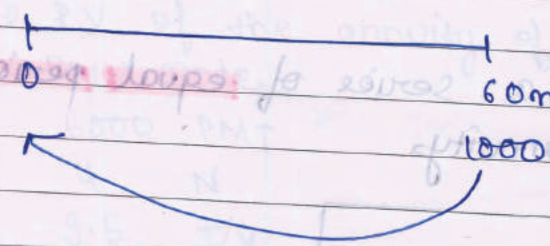
5. -11	$(e^{-0.11} - 1) = -10.416586\%$
6. 18%	$19.721736\,\% \Leftarrow (e^{0.18} - 1)$
7. $\ln(1.24)$ 21.511138%	24%

lord: solve TVM sums at different m

Example ① Consider a 5 year ZCB (Zero Coupon Bond) of FV = 1000

If discount rate is 15% p.a compounded monthly, what should be the price of the ZCB.

→ EPR for 1 month = $\frac{15}{12}$

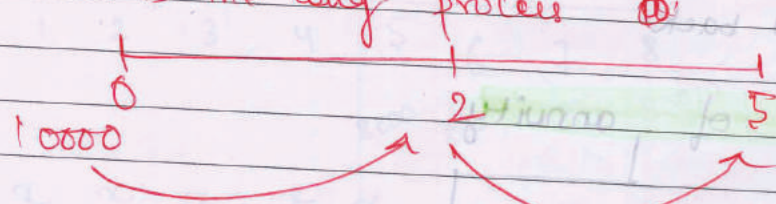


② You deposit $10000 in a bank for 5 years. Bank offers the following interest rates:—

Period	Interest rate
First 2 years	9% p.a. compounded annually
last 3 years	12% p.a. compounded continuously

Calculate the future value of your deposit at the end of 5 years

Note: This is the long process



$10000 \times (1.09)^2 (e^{0.12})^3$

③ Consider a 9month ZCB of face value 1000 if it is presently trading at 917, what is the EAR

→ In the previous sum calculate SAR with m=2 without conversion.

This means 1 period is 6month

917 ± PV
1000 FV
9/6 N
CPT I/Y = x × 2
= 11.8932

Pg-24 Q6 – Q15, Q37 – 39

Los e : Annuity

1) Annuity is a series of equal periodical payments
2) 2 types of annuity

- Ordinary/Regular ⇓ End of Each Period
- Annuity Due ⇓ Beginning of each period

3) PV of annuity

- Ordinary (End): PV comes not at t but at t−1 i.e one period back
- Annuity Due (BGN): PV comes at t

4) FV of annuity

- Ordinary (End): FV comes not at (t−1) but t
- Annuity Due (Bgn): FV comes at (t+1)

633.97
997.37

$250 per quarter — 3 saal baad band
↓→ 8% p.a. compounded quarterly
10 saal baad kitna saath milega

13

5) To set the BGN mode or to remove it —
2nd BGN 2nd set CE|C

6) The periodicity of annuity will decide I/Y — this means if annuity is monthly / quarterly / annually, I/Y has to be the EPR for 1 month / 1 quarter / 1 year

Pg-29
Q(44)

0 1 2 . . . . . . 11 12 13 14

Deposit x x x 6000 6000 6000 6000
Expense

$\frac{8.5}{12} = 0.708\%$ $\frac{9.5}{12} = 0.7916\%$

Step 1 → PV of the annuity of $6000 at t=11
BGN mode
6000 PMT
4 N
$\frac{9.5}{12}$ I/Y
CPT PV

Step 2 → we have END Mode
23718.72 FV
11 N
$\frac{8.5}{12}$ → I/Y
CPT PMT

(48)

0 1 2 3 4 5 6 7 8

8000 8000 8000 8000

x x x x x

11%

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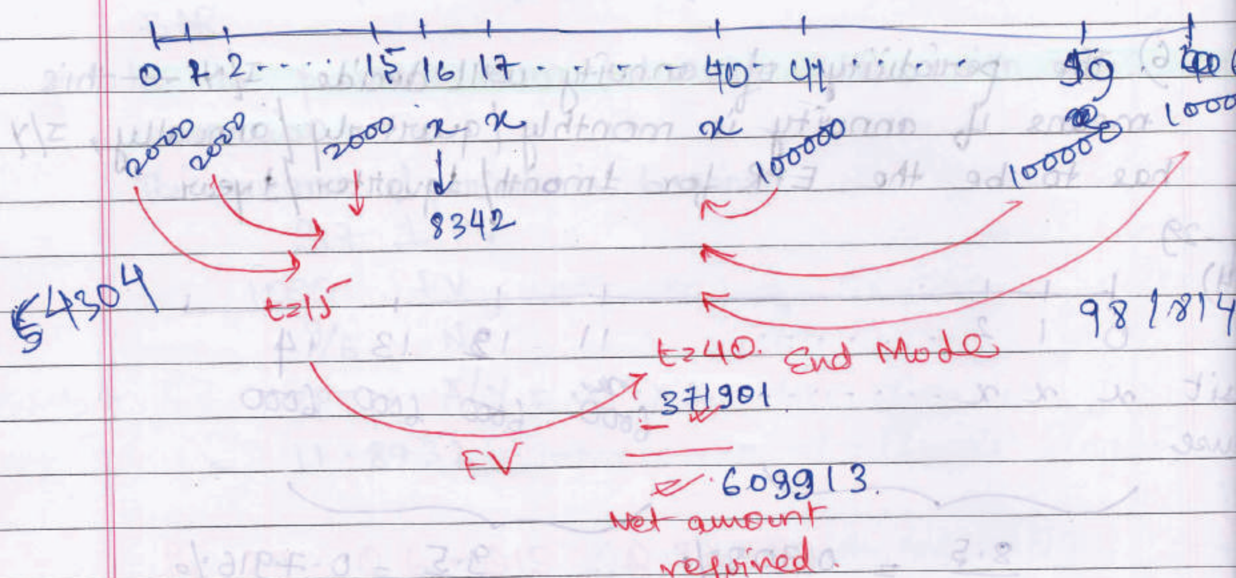
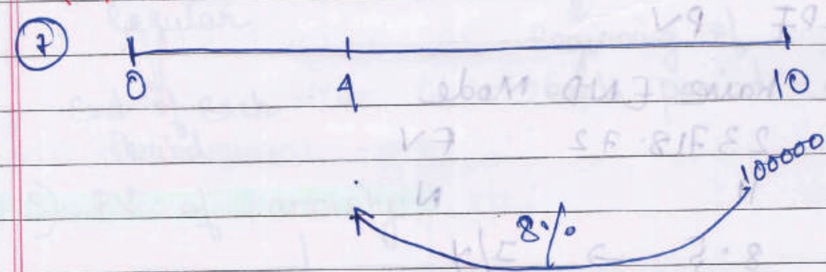
Pg15 Ex-21

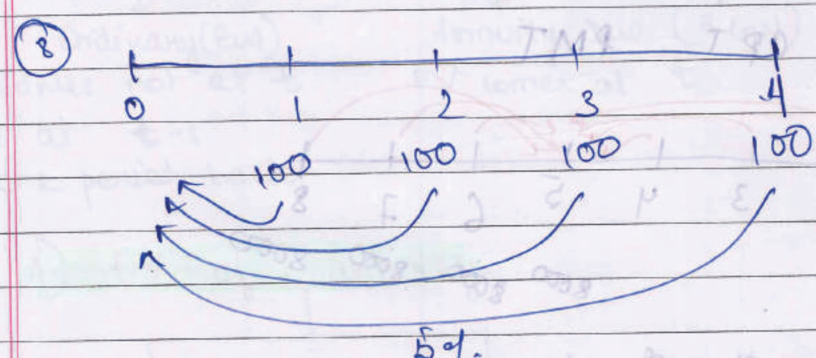


8%

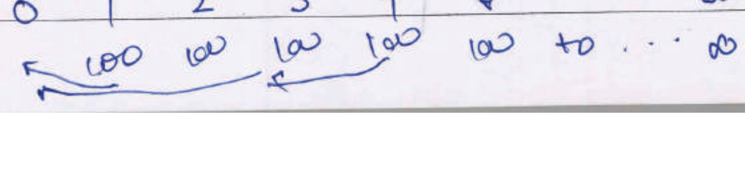
16 peh 15 peh 40 peh 41 peh khare hoke

Pg-21 Q 7





Perpetuity 1



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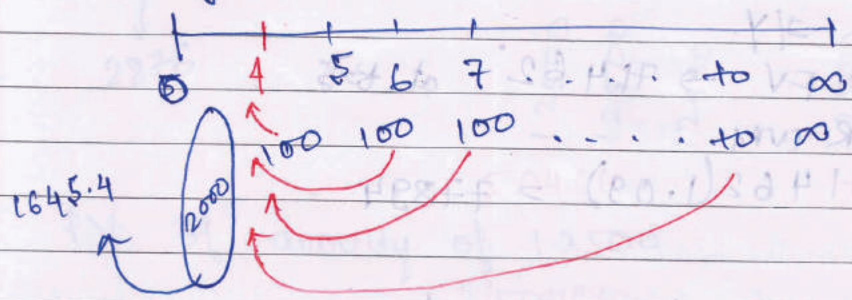
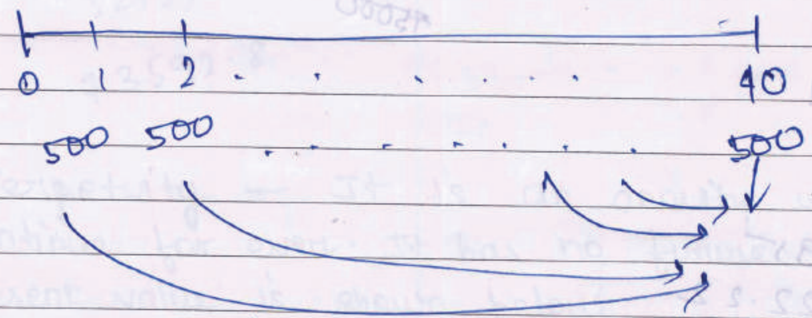
$= \dfrac{100}{0.05} = 2000$

Perpetuity 2



So, $354.6 = 2000 - 1645.4$ [Proved]

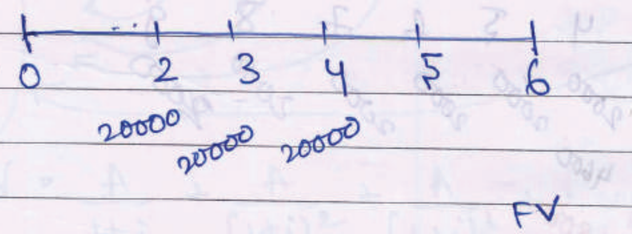
(9)



$33701 - (500 \times 40)$

$= 13701$

(10)



End Mode

20000 PMT

3 N

9 I/Y

CPT $\boxed{FV}$ → at $t=4$

2nd CLR TVM

65562 ± $\boxed{PV}$

2 N

9 I/Y

CPT FV

778942

Page No.

BGN

20000 PMT
3 N
9 I/Y
CPT → FV → 71462 at t=5
2nd CLR TVM
71462(1.09) = 77894

(11)

0 1 2 3 4 5

45000

PV = 56044

(13)
A – 3895
B – 3922.22
C – 3963

(14)

0 1 2 3 4 5 6 7 8 9

2000 2000 2000 2000 2000 2000

4600 4600

1800 1800 1800 1800

PV of the annuity of 2000
BGN – 2000 PMT
PV = 9581.5 at t=4

FV of annuity of 1800
END Mode
~~PV~~ FV = ~~667425~~ 8353.8

0 3 4

4600 4600

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(15)

0 1 2 . . . 10 11 12 13 14

Deposit (x x . . . . . . x)

Expense 2876 12500 2500 12500 12500 12500

50466.

PV of annuity of 12500 = 50466

52585

43592.8

7. Perpetuity — It is an annuity which will continue for ever. It has no future value. Its present value is shown below

0 1 2 3 . . . to ∞

A A A . . . to ∞

$$PV = \frac{A}{1+i} + \frac{A}{(1+i)^2} + \frac{A}{(1+i)^3} + \cdots \text{ to } \infty$$

This is an infinite G.P series with common ratio $r = \frac{1}{1+i}$ and first term i.e $a = \frac{A}{1+i}$

$$\therefore PV = \frac{a}{1-r}$$

$$= \frac{\frac{A}{1+i}}{1-\frac{1}{1+i}} = \frac{A}{i}$$

Answer comes 1 period back just as in ordinary annuity

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Example: ① X Ltd. is a no growth firm. Its DPS = EPS i.e. no retention. If its DPS is expected to be \$40 p.a. forever, what should be the share price at the required rate of return of 10%.

→ $P_0 = \frac{A}{i} = \frac{40}{0.1}$

$= 400$

② A no growth firm presently trading at 500 is expected to pay a DPS of 80 p.a. forever. What is the required rate of return.

→ $P_0 = \frac{A}{i}$

$500 = \frac{80}{i} \Rightarrow i = \frac{80}{500} = 16\%$

8. CASHFLOW MODE (CF Mode) – for unequal cashflow

Example ① What amount needs to be deposited today to recei

— \$400 — 6, 7, 8, 9

— \$500 — 11, 12, 13

\$900 — 15

Discount rate – 7%

→ $PV = \frac{400}{(1.07)^6} + \frac{900}{} + \frac{400}{(1.07)^9} + \frac{500}{(1.07)^{11}} + \cdots + \frac{500}{(1+0.7)^{13}} + \frac{9}{(1.0}$

②
0	1	2	3	4	5
	100	200	300	400	500

rate – 8%

CPT PV

1136.51

③ Mortgage → Loan backed by property

Housing Loan Amt = 2,5000,000

Loan Term = 10 yrs

Interest rate = 12% (it is presumed that th is p.a. compounded monthly)

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(i) Calculate EMI

→ 358677

25000000	± PV
120	N
1	I/Y
CPT	PMT.

(ii) Comment on nature of EMI

→ EMI represent interest inclusive installment

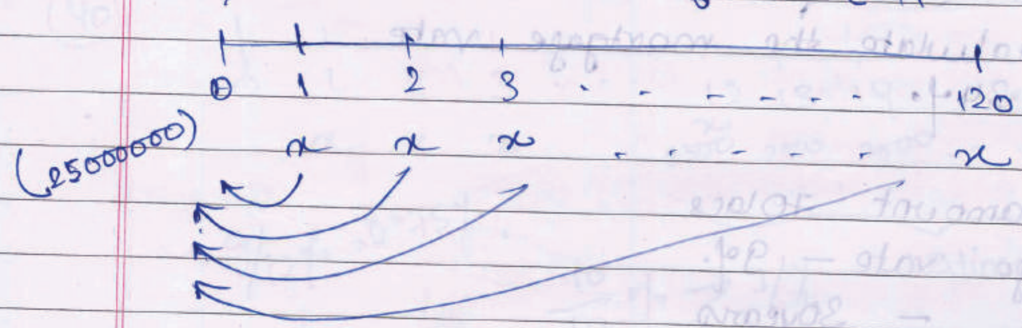
* (iii) Show the equation for calculation of EMI without calculator :

→ $\cancel{A = 25000000(1.12)^{10}}$

Inflow = Outflow

$$25000000 = \frac{x}{1.01} + \frac{x}{(1.01)^2} + \dots + \frac{x}{(1.01)^{120}}$$

i.e, Loan amount = PV of the EMI



(iv) Show the amortisation table

Month	Amount Outstanding	Interest @ 1%	Principal computed	EMI
1	25000000	250000	108677	358677
2	24891323	248913	109764.	"
3	24781559			"
⋮				"
120				"
121	0			0

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(v) compute the interest and Principal component of the 67th EMI

→ After 66 EMI, number of EMI's remaining
= 120 − 66 = 54

∴ loan outstanding after 66 EMI = PV of the remaining 54 EMI's. i.e 358677 PMT

1 I/Y

54 N

CPT PV − 14909724

∴ Interest component of the 67th EMI
= 1% of 14909724
= 149097

Hence Principal component of 67th EMI =
= 358677 − 149097
= 209580 Ans

(vi) (Q) If you took loan of 50 lac for 15 yrs at an EMI of 112000 at the end of each month, calculate the mortgage rate

→ 26.34%

(5) loan amount 70 lacs
Mortgage rate − 9%
Term − 30 years
Calculate Interest and principal component of the 125th installment

→ EMI = 56323.58
Amount outstanding − 6222166.297
Interest component = 46666
Principal component − 9657

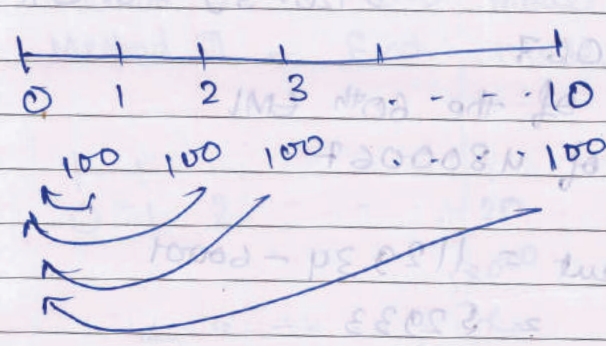
(Q32) 127.45
112.47
92.82

(8) loan term = 10yrs
Mortgage rate = 18%
EMI = 60000
Calculate interest and principal component of the 31^{st} installment.

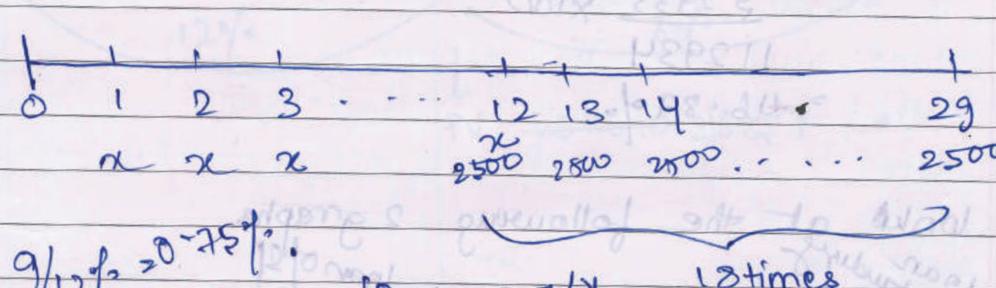
PV 2952591
Interest = 44288
Principal component = 15711

Pg-27 Q32

(34)



(40)



9/12% = 0.75%

$\frac{10}{12}$% → I/Y

(II)　　(I)

Example　Loan Amount = 70lacs
Term = 10yrs　Interest rate = 15%
(i) calculate EMI
→ 112934
(ii) what is the loan outstanding exactly after 3 yrs
→ 5852511
Loan outstanding = Present value of remaining EMI

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(iii) what is the interest and Principal component of the 37th EMI

→ Interest component = 1.25% of 5852511
= 73156

Principal component = 112934 − 73156
= 39778

(iv) What % of EMI is the principal component for the 60th installment

→ Loan outstanding after 59 EMI's = PV of the remaining
= 120 − 59 = 61 EMI

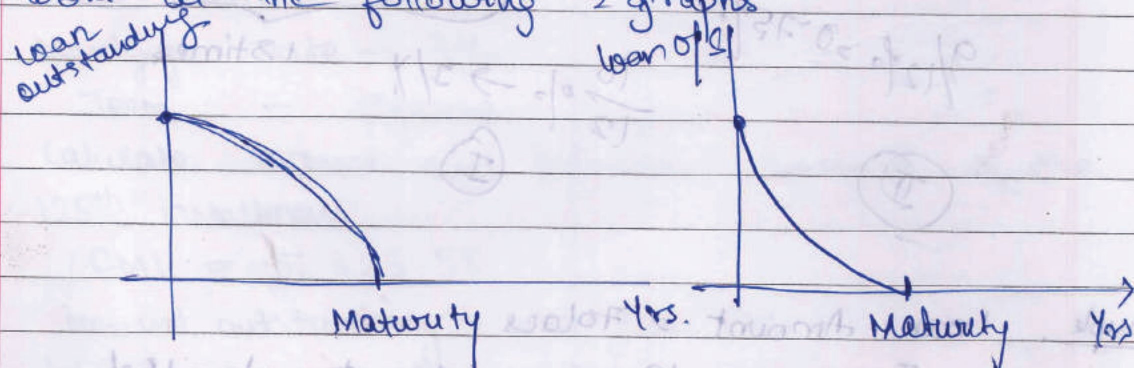
CPT → PV = 4800067

Interest component of the 60th EMI
= 1.25% of 4800067
= 60001

Principal component = 112934 − 60001
= 52933

$\frac{52933}{112934} \times 100$
= 46.87%

Example: look at the following 2 graphs



which graph represents the true picture of loan behaviour in an EMI scheme

The 1st graph is correct — it shows that the loan o/s is decreasing at an increasing rate

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(49) 0 1 2 3 - - - - 10 11 12 13 14

x x x - - - x

12500 12500 12500 12500 12500

Pg - 27 Q 26

PV or EV of annuity due = PV or FV of ordinary annuity X (1+i)

Solve Q 24 - Three ways.

Method I - BGN Mode

Method II - END Mode N = 11 + 5000

Method III - End Mode N = 12 X (1.075)

(50) 0 1 2 - - - 20 - - - 50

18000 - - - - - 18000

x x - - x

12% 8%

PV = ~~202640~~ 218851