SSEI
Sanjay Saraf Educational Institute Pvt. Ltd.
A positive NPV Training Center

CFA LEVEL 1

MARATHON SERIES

QUANTITATIVE METHODS

Question 1:

A retiree will begin taking withdrawals of NOK 50,000 per year from an investment account 15 years from today. The payments will last for 20 years. The retiree also wants to have NOK 100,000 remaining in the account at the end of the withdrawal period. If the discount rate is 7%, the lump sum (in NOK) that must be invested today to satisfy both retirement goals is closest to:

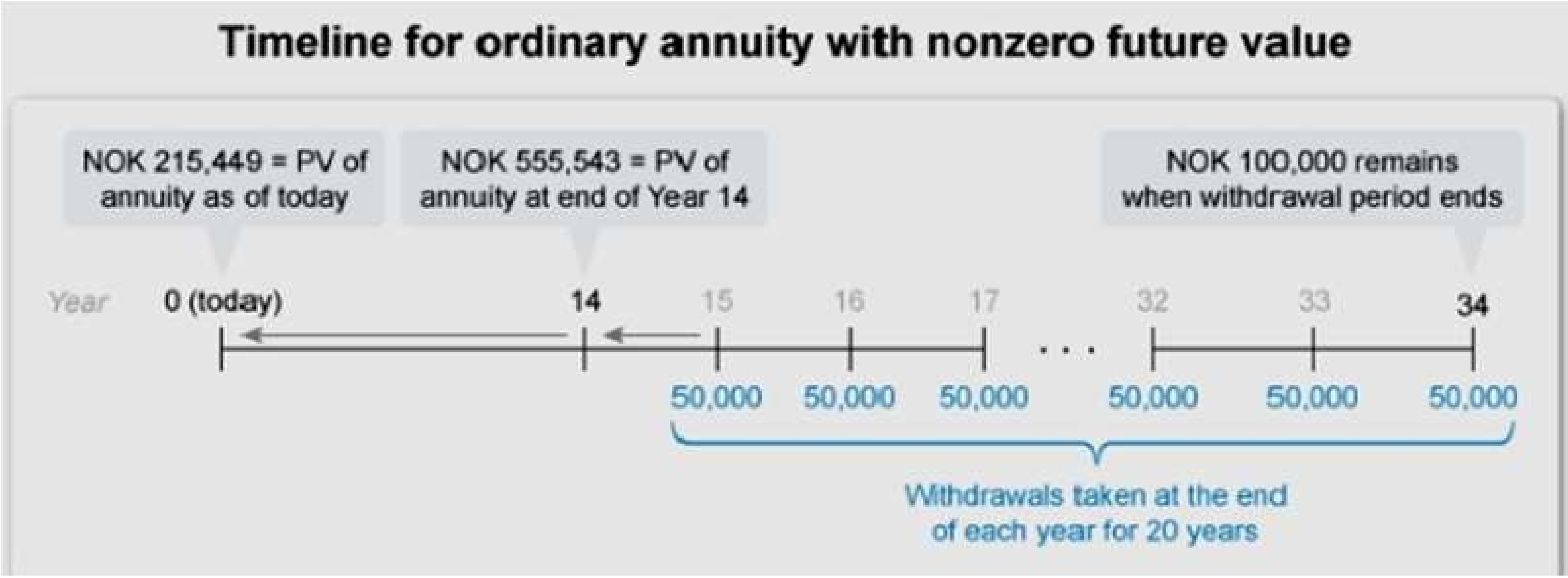
A. 201,354

B. 215,449

C. 555,543

Solution:

B is correct.



This question require two calculations:

The amount needed in an account at the beginning of Year 15 (ie, the end of Year 14)to fund annual NOK50,000 withdrawals for 20 years and have NOK100,000 remaining at the end of the annuity period(ie, the end of Year 34) ;and The amount that must be deposited today to fund the annuity.

As the timeline shows, the present value of the annuity at the end of Year 14 is NOK555,543. This is the amount needed to fund 20 annual withdrawals of NOK 50,000,with the first withdrawal occurring at the e of Year 15, and to have NOK 100,000 in the account at the end of the annuity period. Discounting NOK 555,543 over 14 years at a discount rate of 7% produces a present value of approximately NOK215,449.

A is incorrect. NOK 201,354 results from calculating today's present value of the annuity using 15 years instead of 14.

C is incorrect. NOK 555,543 is the present value of the annuity at the end of Year 14.The lump sum to be deposited today that produces NOK555,543 is NOK215,449.

Question 2:

Which of the following statements is most accurate with respect to the chi-square test?

A. The population must be normally distributed.

B. Valid inferences from the test can be based on nonrandom samples.

C. The test statistic is based on the ratio of the population variance to the sample variance.

Solution:

A is correct.

The chi-square test (ie, X2) tests whether the variance of a sample drawn from a population represents the variance of that population. An example is a sample of 200 monthly returns of a fund that has a variance of 0.0256. The chi-square test allows the analyst to determine whether the sample variance is representative of the overall variance of the fund.

For conclusions from a chi-square test to be valid, both of the following conditions must be satisfied:

The population must be normally distributed.

The sample must be randomly drawn from the population (Choice B).

The chi-square test can be either one- or two-tailed. It is performed by comparing the variance of a sample drawn from the population to the hypothesized variance of the population. The chi-square test statistic is given as:

(Choice C) As shown by the chi-square test, the numerator is the sample variance and the denominator is the population variance.

Note:

The chi-square test is used to test hypotheses concerning the variance of a population. For conclusions based on the test to be valid, the population must be normally distributed and the sample must be randomly drawn. The test statistic is generated by multiplying the ratio of the sample variance to the hypothesized population variance by (n − 1), where n is the number of observations in the sample.

Question 3:

An analyst performs hypothesis testing on a normal distribution. If the distribution's p-value is 0.01, most likely:

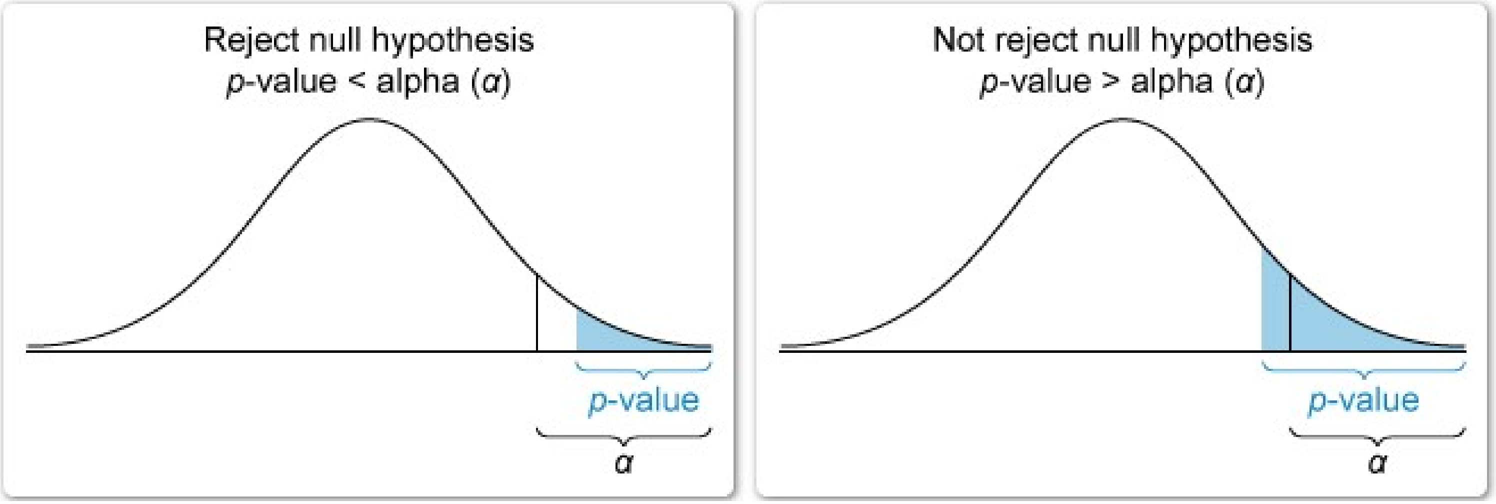
A. there is a 1% probability of a Type II error.

B. the null hypothesis can be rejected at the 95% confidence level.

C. for a two-tailed test, the null rejection point is approximately 2.33 standard deviations from the mean.

Solution:

B is correct.



The p-value is the lowest level of significance (shown as α in the images above), where the null hypothesis can be rejected. In this question, a p-value of 0.01 means that the null should be rejected if the hypothesis is tested at a significance of 0.01. A confidence level for a test equals 1 − the level of significance, so a 95% confidence level is the same as a significance level of 0.05.

If the p-value is less than a specified level of significance, the null should be rejected. In this question, 0.01 is less than 0.05, so the correct decision is to reject the null at the 95% confidence level.

(Choice A) The significance level of the test equals the probability of a Type I error, not a Type II error.

(Choice C) For hypothesis tests on a normal distribution, a two-tailed hypothesis test with a 0.01 significance level results in a null rejection point of approximately 2.58. This represents a value that is ± 2.58 standard deviations from the mean. The rejection point for a one-tailed test at a 0.01 significance level is approximately 2.33.

Note:

The p-value represents the smallest level of significance at which a null hypothesis can be rejected. When a p-value is less than a specified level of significance, the correct decision is to reject the null hypothesis.

Question 4:

For a given series of returns, the variable that will cause the largest difference between the returns' arithmetic mean and geometric mean is:

A. wider range.

B. greater variance.

C. more observations.

Solution:

B is correct.

Approximate relationship between geometric and arithmetic means

$$\text{Mean}_{\text{geometric}} \approx \text{Mean}_{\text{arithmetic}} - (0.5 \times \text{Variance})$$

For any series of returns, the arithmetic mean is always equal to or greater than the geometric mean. Uncertainty, reflected by the variance of dispersion of returns around a mean, causes the arithmetic mean to exceed the geometric mean.

For example, the returns for the past three years for Funds A and B are:

Year	1	2	3
Fund A	−3%	4%	8%
Fund B	2%	3%	4%

The arithmetic mean for both funds is 3%. The geometric mean for each fund is as follows:

$$\begin{aligned} \text{Geometric}_A &= \left[(1 - 0.03) \times (1 + 0.04) \times (1 + 0.08)\right]^{1/3} - 1 \\ &= 0.0290 \\ &= 2.90\% \end{aligned}$$

$$\begin{aligned} \text{Geometric}_B &= \left[(1 + 0.02) \times (1 + 0.03) \times (1 + 0.04)\right]^{1/3} - 1 \\ &= 0.0299 \\ &= 2.99\% \end{aligned}$$

Fund A's variance is much larger than that of Fund B (0.00207 vs. 0.00007). The greater variance corresponds to the greater difference between each fund's arithmetic and geometric mean.

The formula above provides an estimate for the geometric mean when the arithmetic mean is known. For Fund A, the geometric mean is estimated as 0.03 − (0.5 × 0.00207) ≈ 0.0289, or 2.89%, which is very close to the calculated geometric mean of 2.90%.

(Choice A) The range of returns in a distribution identifies the lowest and highest observations but does not indicate the dispersion of returns.

(Choice C) It is the dispersion in the series of observations, not the number of observations in the series, that affects the difference between the arithmetic and geometric means. A series with many observations, but a small variance, will result in a small differential between the two means. By contrast, a series with fewer, but more dispersed, observations will have a larger differential.

Note:

The arithmetic mean for a series of returns is always equal to or greater than the geometric mean. Greater uncertainty in returns results in a greater difference between the two means. Variance, which measures dispersion of returns around a mean, reflects that uncertainty and accounts for the difference between the means.

Question 5:

An analyst performs a hypothesis test on the correlation of monthly returns of Fund A and Fund B. The null hypothesis is that the returns are uncorrelated with each other. Increasing the number of observations used in the test most likely results in an increase in the:

A. critical value.

B. probability of rejecting a true null hypothesis.

C. probability of rejecting a false null hypothesis.

Solution:

C is correct.

t-test for correlation

$$t = r \times \frac{\sqrt{n-2}}{\sqrt{1-r^2}}$$

r = correlation coefficient
n = number of observations

A hypothesis test on correlation based on sample observations assesses whether correlation in fact exists or is equal to zero (ie, no correlation). The hypothesis test for correlation is a two-tailed t-test. The null hypothesis is that correlation equals zero with the alternative that correlation is unequal to zero, or:

$$H_0: r = 0 \text{ vs. } H_A: r \neq 0$$

The test statistic is compared to a critical value in a t-distribution table with degrees of freedom (df) equal to n – 2. The critical value is based on the test's level of significance (eg, 0.01, 0.05). If the absolute value of the test statistic is higher than the critical value, the null hypothesis of no correlation is rejected in favor of the alternative that correlation exists.

Increasing the sample size n results in two consequences:

- The critical value decreases
- The test statistic increases

The effect of both consequences is to increase the likelihood that the test statistic will exceed the critical value. This leads to a higher probability of rejecting a null hypothesis that is false (ie, making a correct decision).

(Choice A) Increasing the sample size makes the critical value lower, not higher.

(Choice B) As the sample size increases, the probability of rejecting a false null hypothesis increases, so the probability of rejecting a true null hypothesis (ie, a Type I error) decreases.

Question 6:

Mike Far explains the linear regression model and its underlying assumptions using the following statement: “The estimated parameters in a linear regression model maximize the sum of the squared regression residuals.” The above statement on estimated parameters in a linear regression model is most likely

A. correct

B. incorrect, because the model minimizes the sum of squared regression residuals.

C. incorrect, because the model minimizes the sum of the regression residuals.

Solution:

B is correct.

A linear regression model computes a line that best fits the observations. It chooses values for the intercept and slope that minimize the sum of the squared vertical distance between the observations and the regression line. Hence, the estimated parameters in a linear regression model minimize the sum of the squared regression residuals.

Question 7:

A researcher has developed a linear regression model to predict the sales of a company based on the advertising expenses. The model has an R-squared value of 0.75. What does this value indicate about the model?

A. The model explains 75% of the variation in sales.

B. The model explains 75% of the variation in advertising expenses.

C. The model is not a good fit because the R-squared value is too low.

Solution:

A is correct.

The R-squared value measures the proportion of variation in the dependent variable that is explained by the independent variable(s) in the model. An R-squared value of 0.75 indicates that 75% of the variation in sales can be explained by the variation in advertising expenses in the model. Therefore, option A is correct.

Option B is incorrect because R-squared measures the variation explained by the independent variable(s), not the dependent variable.

Option C is incorrect because an R-squared value of 0.75 indicates a relatively good fit for the model. The closer the R-squared value is to 1, the better the model fits the data.

Question 8:

A large positive value of the Spearman rank correlation such as 0.90 would most likely indicate that:

A. a high rank in one year is associated with a low rank in the second year.

B. a high rank in one year is associated with a high rank in the second year.

C. a high rank in one year will not have any impact on the rank in the second year.

Solution:

B is correct.

A large positive value of the Spearman rank correlation such as 0.90 would most likely indicate that a high rank in one year is associated with a high rank in the second year.
Note: In statistics, Spearman's rank correlation coefficient or Spearman's rho is a non-parametric measure of statistical dependence between two variables. It takes values from -1 to 1. The closer to 1 or to -1, the stronger the relationship. 1 indicates a perfect positive association/relationship, whereas -1 indicates a perfect negative association between variables.

Question 9:

A market research firm wants to conduct a survey to gather data on consumer preferences for a new product. They plan to select a sample of 500 people from a population of 10,000 potential consumers. Which of the following sampling methods would best ensure that each person in the population has an equal chance of being included in the sample?

A. Quota sampling

B. Stratified sampling

C. Random sampling

Solution:

C is correct.

Random sampling would best ensure that each person in the population has an equal chance of being included in the sample. In random sampling, each individual in the population has an equal chance of being selected, which helps to minimize bias in the selection process. This method is useful when the population is homogeneous, and each member has an equal chance of being included in the sample. Therefore, the answer is C.
Quota sampling, as in option A, involves selecting participants based on specific characteristics, such as age or gender, to ensure that the sample matches the population's demographic profile. However, this method can be biased if the sample is not representative of the population.

Stratified sampling, as in option B, involves dividing the population into homogeneous subgroups, or strata, and then randomly selecting participants from each stratum. This method is useful when there are differences within the population, but it may not ensure that each person in the population has an equal chance of being included in the sample.

Question 10:

Natalie Brunswick, neurosurgeon at a large U.S. university, was recently granted permission to take an 18-month sabbatical that will begin one year from today. During the sabbatical, Brunswick will need $2,500 at the beginning of each month for living expenses that month. Her financial planner estimates that she will earn an annual rate of 9% over the next year on any money she saves. The annual rate of return during her sabbatical term will likely increase to 10%. At the end of each month during the year before the sabbatical, Brunswick should save approximately:

A. $3,505.00

B. $3,356.00

C. $3,330.00

Solution:

B is correct.

This is a two-step problem. First, we need to calculate the present value of the amount she needs over her sabbatical. (This amount will be in the form of an annuity due since she requires the payment at the beginning of the month.) Then, we will use future value formulas to determine how much she needs to save each month(ordinary annuity).

Step 1: Calculate present value of amount required during the sabbatical

Using a financial calculator:

Set to BEGIN Mode, then N = 12 × 1.5 = 18; I/Y = 10 / 12 =0.8333;PMT=2,500;FV =0;CPT→ PV =41,974

Step 2: Calculate amount to save each month

Make sure the calculator is set to END mode, then N = 12; I/Y = 9 / 12 = 0.75; PV = 0;FV=41,974; CPT→ PMT = -3,356

Question 11:

Which of the following expressions best describes the relationship between the nominal rate of return, the real risk-free rate of return, and the inflation rate that can be used in hyperinflationary markets?

Equation I: (1 + Nominal rate) = (1 + Real risk - free rate of return) × Inflation rate.

Equation II: Nominal rate =Real rate + Inflation rate.

Equation III: (1 + Nominal rate) = (1 + Real risk - free rate of return) × (1 + Inflation rate).

A. Equations I, II, and III.

B. Equations II and III.

C. Equation III.

Solution:

C is correct.

Technically, 1 plus the nominal rate equals the product of 1 plus the real rate and 1plus the inflation rate. As a quick approximation, however, the nominal rate is equal to the real rate plus an inflation rate. However, Equation II cannot be used in the hyperinflationary markets.

Question 12:

Harlan Porterfield, CFA, is conducting the price analysis of a 5% bond maturing after five years. The current market price of the bond is $978.65, and the market interest rate is 5.50%. The probability of the market interest rate remaining stable is 0.40, while the probability of the market interest rate decreasing to 5.25% is 0.60. He puts forth the following possibilities:

- If the market interest rate remains stable, the bond has a probability of 0.80 for the market price remaining the same and a probability of 0.20 for a market price of $965.00.
- If the market interest rate decreases to 5.25%, the bond has a probability of 0.90for properly reflecting the new market interest rate and a probability of 0.10 for not reacting to the change in the interest rate.

Assuming the estimates provided are accurate, the standard deviation of values under the stable interest rate scenario as compared to the declining interest rate scenario is

A. higher

B. lower

C. the same

Solution:

A is correct.

First, we need to calculate the bond price if the market price decreases : N=5; I/Y= 5.25;PMT=50;FV =1000; CPT→ PV =989.25

Expected bond price given stable interest rate:

E(Bond price|stable interest rate)
=0.80($978.65)+0.20($965.00)
=$975.92

Expected bond price given declining interest rate:

E(Bond price|declining interest rate)

=0.90($989.25)+0.10($978.65)=$988.19

Now, we can calculate the expected bond price:

σ^2(Bond price|stable interest rate)

=0.80[($978.65–$975.92)^2]+0.20[($965.00–$975.92)^2]=29.8116

σ^2(Bond price|declining interest rate)

=0.90[($989.25–$988.19)^2]+0.10[($978.65

–$988.19)^2]=10.1137

Therefore, the standard deviation of values is higher under a stable interest rate scenario compared to a declining interest rate scenario

Question 13:

According to research, heart attack is one of the leading causes of death world wide with about 7.6 million casualties in 2013.There search also claimed that“ male smokers are likely to experience heart attack 15 times more than those who don't smoke and female smokers are likely to experience heart attack 12 times more than nonsmoking females.” Furthermore, the National Statistics Office released data that 16% of females worldwide are smokers. Suppose you're in a hospital, waiting for your turn for a medical checkup, and the female beside you is having a monthly maintenance checkup following a heart attack. What are the chances that she is a smoker?

A. 69.57%

B. 16.24%

C. 27.61%

Solution:

A is correct.

P(S) =16%, thus, P(NS)=84%

P(H/S) =12×P(H/NS), thus, P(H/NS)=P(H/S)/12

Also,

P(H) =P(H&S)+P(H&NS)

=P(H/S)xP(S)+P(H/NS)/P(NS)

=P(H/S)xP(S)+(P(H/S)/12)/P(NS)

We want P(S/H) which equals :

P(H&S)/P(H) =(P(H/S)xP(S))/(P(H/S)xP(S)+(P(H/S)/12)/P(NS))

=P(S)/(P(S)+P(NS)/12)

=16%/(16%+84%/12) = 0.6957

Question 14:

As a member of the forensics team in a murder case, you found a DNA sample which is that of the murderer's and there is a 0.2% chance that a random person's DNA matches this sample. In your search, one of the alleged suspects' DNA matches with the sample you have obtained. You know that there is only a .004% chance that a random person is the actual murderer but there's a 100% chance that the man's DNA matches that of the murderer's given that the man is indeed the one who killed the victim. What are the chances that the man is proven guilty given that his DNA matches your sample?

A. 5%

B. 2.5%

C. 2%

Solution:

C is correct.

The third choice is correct.

P(A) = .004%

P(B) = .2%

P(B|A) = 1

P(A|B) = P(B|A) × P(A) / P(B)

P(A|B) = 1 × .00004/.002 P(A|B) = 2%

Question 15:

A portfolio manager is analyzing the returns of a stock over the past year. The manager wants to understand the distribution pattern of the returns to select the appropriate statistical tools for analysis. Which of the following visualizations would best help the manager to understand the distribution pattern of the stock returns?

A. Bar chart

B. Scatter plot

C. Box plot

Solution:

C is correct.

A box plot is the best visualization to understand the distribution pattern of the stock returns. A box plot summarizes the distribution of a dataset by showing the median, quartiles, and outliers. It also provides information about the skewness of the distribution and the presence of outliers. This information is useful for selecting appropriate statistical tools for analysis.

A bar chart, as in option A, is used to compare the values of different categories. It is not appropriate for analyzing the distribution of a single variable.

A scatter plot, as in option B, is used to visualize the relationship between two variables. It is not appropriate for analyzing the distribution of a single variable.
Therefore, the answer is C.

Question 16:

A researcher used a chi-square test to test the null hypothesis of H : $\sigma^2 \leq 100$ at alpha= 5%. With 33 degrees of freedom, the critical value associated with 5% in the left tail is 11.6885 and the critical value associated with 5% in the right tail is 38.07563. The test statistic is 5.43. The researcher will:

A. reject the null since 5.43 < 11.6885.

B. reject the null since 5.43 < 38.07563.

C. fail to reject the null since 5.43 < 38.07563.

Solution:

C is correct.

Based on the null, we can see that this is a right-tailed test. Thus, the critical value of concern is 38.07563. Since 5.43 < 38.07563, we fail to reject the null.

Question 17:

Which statement is incorrect regarding a X^2distribution?

A. It is the distribution of variances
B. It is the sum of independent (z values)
C. It is the distribution of a sum of squares of independent standard normally distributed random variables

Solution:

A is correct.

First, a standardized normal variable has the form $(X_i - \bar{X})/\sigma$.

Squaring results in
$\left[(X_i - \bar{X})/\sigma\right]^2 = (z - value)^2$.

A χ^2 distributed variable is the
$$\sum \left[(X_i - \bar{X}/\sigma\right]^2 = \sum (z - value)^2$$
The term in square brackets is the sum of squares of standard normal variables.

Question 18:

An analyst wants to investigate the relationship between CEO compensation and firm performance in the technology industry. The analyst randomly selects 100 companies from a list of all technology companies and divides them into four subgroups based on their market capitalization: small, medium, large, and mega-cap. The analyst then randomly selects 25 companies from each subgroup to create a sample of 100 companies. The analyst has selected a:

A. cluster sample.

B. non-probability sample.

C. stratified random sample.

Solution:

C is correct.

This is an example of a stratified random sample, where the population is divided into subgroups based on certain characteristics and then a random sample is selected from each subgroup. The goal is to ensure that each subgroup is proportionally represented in the sample to provide a more representative sample of the population.

Question 19:

Mike's dream is to be able to withdraw $210,000 at the start of each year when he finally retires 15 years from now. However, he wants to leave his family some money amounting to $1,800,000 by the time he passes away, which he expects to be 40 years from now. To do this, Mike plans to work hard at his law firm to be able to invest some of his earnings at the end of each year at a rate of 10%. He also plans to transfer his investment when he retires to more secure alternatives that will yield him an 8% return annually. How much should Mike invest yearly for 15 straight years to fulfill his dreams?

A. $84,471.56

B. $184,337.45

C. $210,000.00

Solution:

A is correct.

PV of Withdrawals = [Yearly withdrawal]×[PVAF(annuity due) 8% for 25periods]
= 210,000 × 11.5288 =2,421,039

PV of Lumpsum = [Lumpsum]/[PV of one 8% for 25 periods]
$=1,800,000/1.08^{25}$
=262,832.23

FV of desired amount after retirement =$2,421,039.00 +$262,832.23 =$2,683,871.47

[Yearly investment] = [FV of desired amount after retirement]/[FVAF 10% for 15 periods]
=$2,683,871/31.7724
=$84,471.56